Physical and Numerical Modeling of Dissection Propagation in Arteries caused by Balloon Angioplasty

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Abstract. Arterial dissections caused by balloon angioplasty have been implicated as a contributing factor to both acute procedural complications and chronic restenosis of the treatment site. However, no related biomechanical studies are known in the literature. The mechanical properties of the arterial wall are controlled by the rubber-like protein elastin, fibrous protein collagen and smooth muscle cells. In the media of elastic arteries these constituents are found in thin layers that are arranged in repeating lamellar units and favor dissection type of failure. The presented approach models the dissection of the media by means of strong discontinuities and the application of the theory of cohesive zones. Thereby, the dissection is regarded as a gradual process in which separation between incipient material surfaces is resisted by cohesive traction. The applied numerical frame is based on the Partition of Unity Finite Element Method (PUFEM) and has been utilized for tetrahedral elements. A tracking algorithm for 3D non-planar cracks captures the evolution of multiple non-interacting dissections. The proposed concept is applied to investigate the dissection of the media due to balloon angioplasty, where the associated material parameters are determined from failure experiments on human tissue.

1 Introduction

Arterial dissection phenomena are frequently observed in clinical practice, e.g., during balloon angioplasty [1], an established and effective therapeutic intervention to reduce the severity of atherosclerotic stenosis. Balloon angioplasty involves denudation of endothelium, disruption of the intima and the atherosclerotic plaque with frequent separation from or dissection of the media, and overstretching of non-diseased portions of the arterial wall, see, e.g., [2] and references therein. In particular, dissection is a characteristic form of arterial trauma involving laceration and/or cleavage of the arterial wall. Plaque fracture and/or dissections are major contribution to the gain in lumen due to balloon angioplasty intervention [1].

Moreover, dissection has been implicated as a contributing factor to both acute procedural complications (abrupt reclosure, ischemia, myocardial infarction, emergency surgery, and coronary microembolization [3–7]), and chronic restenosis of the treatment site [8]. Maximal luminal gain, amount of controlled injury and the risk of wall fracture are quantities to be optimized for the considered interventional procedure.

The rubber-like protein elastin, the fibrous protein collagen and smooth muscle cells control the mechanical properties of arterial walls. In particular, in the media of elastic arteries these three components are found to be organized in medial lamellar units [9], each of which about 10µm thick [10]. Consequently, the laminated structures are prone to open up creating a cleavage plane between lamellae [11]. Moreover, the organized structure of elastin, collagen and smooth muscle cells [10] leads to the experimental observed anisotropic (local cylindrical orthotropic) response [12–14] of the arterial wall.

In the present work we capture the kinematics of
the dissection process by means of discontinuities in the displacement field, i.e. the strong discontinuity approach is pursued. Moreover, a fracture process zone is introduced and the dissection is regarded as a gradual process in which separation between incipient material surfaces is resisted by cohesive traction. We employ a (discrete) constitutive description of the cohesive zone, which is based on a transversely isotropic Traction Separation Law (TSL) of exponential type with isotropic damage. The formulation is based on the theory of invariants [15] and non-negativeness of the damage dissipation is guaranteed. For more details on the chosen approach see [16].

The introduced constitutive concept is numerically represented within the PUFEM, [17], and has been utilized for tetrahedral elements throughout this work. Moreover, a tracking algorithm for 3D non-planar cracks characterizes the evolution of multiple non-interacting dissections. The proposed numerical concept leads to an efficient and robust finite element formulation suitable for failure analyses [18]. In the present work this concept is applied to investigate dissection type of failure in the media caused by balloon angioplasty. Therefore, material parameters characterizing the dissection properties of the human media are determined from failure experiments of human tissue.

2 Continuum framework

In this section the underlying continuum mechanical relations and the concept of strong discontinuity are reviewed briefly; a more detailed derivation can be found in, e.g., [19, 16]. In particular, the discrete kinematics, the variational formulation and the discrete constitutive model are discussed therein.

2.1 Discrete kinematics

We consider a body, where an embedded discontinuity \(\partial \Omega_d\) separates its reference configuration \(\Omega_0\) into two sub-domains \(\Omega_{0+}\) and \(\Omega_{0-}\). The deformation \(\chi(X)\) maps \(\Omega_{0+}\) and \(\Omega_{0-}\) into their current configurations \(\Omega_{+}\) and \(\Omega_{-}\), where \(X\) denotes the referential coordinate of a material point. The kinematics of the dissection is then represented by the introduction of a discontinuity in the displacement field, i.e. \(u(X) = u_c(X) + \mathcal{H}(X)u_e(X)\), where \(\mathcal{H}\) denotes the Heaviside function. The smooth fields \(u_c\) and \(u_e\) denote compatible and enhanced displacements, respectively.

Consequently, the material displacement gradient reads

\[
\text{Grad} u(X) = \text{Grad} u_c(X) + \mathcal{H}\text{Grad} u_e(X) + \delta_d(X)u_e(X) \otimes N(X_d),
\]

where we employed the property \(\mathcal{H}(X) = \delta_d N(X_d)\) of the Heaviside function and \(\delta_d\) denotes the Dirac-delta functional centered at the discontinuity. Moreover, the referential unit direction vector \(N(X_d)\) characterizes the orientation of the embedded discontinuity \(\partial \Omega_d\) at the material point \(X_d\).

With eq. (1) the deformation gradient \(F = I + \text{Grad} u\) is then defined. It serves as a basis for the right \(C = F^TF\) and left \(b = FF^T\) Cauchy Green strain tensors [20, 21].

The representation of the mechanical response of the dissection by means of a cohesive zone model, to be discussed subsequently, requires the introduction of a fictitious discontinuity \(\partial \Omega_d\), which is located in the current configuration. We follow [18] and define \(\partial \Omega_d\) via the associated deformation gradient \(F_d(X_d) = I + \text{Grad} u_c + u_e \otimes N/2\). The factor 1/2 enforces \(\partial \Omega_d\) to be placed in the middle between the two (physical) surfaces created by the dissection. Consequently, the unit normal vector \(n = NF_d^{-1}/|NF_d^{-1}|\) onto \(\partial \Omega_d\) is defined by a weighted push-forward of the covariant vector \(N\).

2.2 Variational formulation

We start with the standard single-field variational principle and neglect inertia effects such that \(\int_{\Omega_0} \text{Grad} \delta u : P(F)dV - \delta \Pi^{\text{ext}}(\delta u) = 0\) holds [20, 21], where \(\Pi^{\text{ext}}\) is the virtual external potential energy, and \(P\) and \(\delta u = \delta u_c + \mathcal{H}\delta u_e\) denote the first Piola-Kirchhoff stress tensor and the admissible variation of the displacement field, respectively. Consequently, the single-field variational principle is provided in form of two spatial variational statements [16], i.e.
\[
\begin{align*}
\int_{\tilde{\Omega}_c} \text{sym}(\text{grad}_c \delta \mathbf{u}_c) : \mathbf{\sigma}_c dv + \int_{\tilde{\Omega}_c} \text{sym}(\text{grad}_c \delta \mathbf{u}_c) : \mathbf{\sigma}_c dv - \delta \Pi_c^{\text{ext}}(\delta \mathbf{u}_c) &= 0, \\
\int_{\tilde{\Omega}_c} \text{sym}(\text{grad}_c \delta \mathbf{u}_c) : \mathbf{\sigma}_c dv + \int_{\partial \tilde{\Omega}_d} \mathbf{t} \cdot \delta \mathbf{u}_c ds - \delta \Pi_c^{\text{ext}}(\delta \mathbf{u}_c) &= 0,
\end{align*}
\]

where \(dv\) and \(ds\) are the spatial volume and surface elements, respectively. Moreover, \(\mathbf{\sigma}_c = J_c^{-1} \mathbf{P}(\mathbf{F}_c) \mathbf{F}_c^T\) and \(\mathbf{\sigma}_c = J_c^{-1} \mathbf{P}(\mathbf{F}_c) \mathbf{F}_c^T\) denote the Cauchy stress tensors and \(\mathbf{t} = \mathbf{T} dS/ds\) is the Cauchy traction vector associated with a fictitious discontinuity \(\partial \Omega_d\). The spatial gradients in (2) are defined according to \(\text{grad}(\bullet) = \text{Grad}(\bullet) \mathbf{F}_c^{-1}\) and \(\text{grad}(\bullet) = \text{Grad}(\bullet) \mathbf{F}_c^{-1}\), and \(\text{sym}(\bullet) = ((\bullet) + (\bullet)^T)/2\) furnishes the symmetric part of \((\bullet)\). We have also assumed dead loads and introduced the split \(\delta \Pi_c^{\text{ext}} = \delta \Pi_c^{\text{ext}} + \delta \Pi_c^{\text{ext}}\).

The variational statement (2) and the related consistent linearization are the basis for the PUFEM implementation of the proposed dissection model [16].

### 2.3 Discrete constitutive model

In the variational statement (2), the constitutive response of the cohesive zone is represented by the Cauchy traction vector \(\mathbf{t}\). We assume a transversely isotropic cohesive zone, where the preferred direction is defined by the normal \(\mathbf{n}\) onto the fictitious discontinuity \(\Omega_d\) and the state of damage is represented by a scalar internal variable \(\delta \in [0, \infty[\) such that \(\psi = \psi(\mathbf{u}_c \otimes \mathbf{u}_c, \mathbf{n} \otimes \mathbf{n}, \delta)\). Following the theory of invariants [22], the cohesive potential reads \(\psi = \psi(i_1, i_2, i_3, i_4, i_5, \delta)\), where \(i_1, \ldots, i_5\) are the invariants associated with the symmetric second-order tensors \(\mathbf{u}_c \otimes \mathbf{u}_c\) and \(\mathbf{n} \otimes \mathbf{n}\). The cohesive potential is parameterized as

\[
\psi(i_1, i_4, \delta) = \frac{t_0}{2b} \exp(-a \delta^b)[i_4 + \alpha(i_1 - i_4)],
\]

where \(t_0, a, b, \) and \(\alpha\) are (non-negative) material parameters to be determined from experimental data. In particular, \(t_0, a, b\) characterize the dissection strength and the softening response of the cohesive zone, respectively. Moreover, \(\alpha\) denotes the ratio between the transverse and normal stiffness of the cohesive zone, and hence determines its anisotropy. Note that eq. (3) assumes that the cohesive potential depends solely on \(i_1 = \mathbf{I} : (\mathbf{u}_c \otimes \mathbf{u}_c)\) and \(i_4 = (\mathbf{u}_c \otimes \mathbf{u}_c) : (\mathbf{n} \otimes \mathbf{n})\).

In order to complete the model we define the damage surface \(\phi(\mathbf{u}_c, \delta) = |\mathbf{u}_c| - \delta = 0\) in the three-dimensional enhanced displacement space and assume \(\delta = |\mathbf{u}_c|\) to describe the evolution of the internal (damage) variable \(\delta\). Finally, according to the procedure by COLEMAN and NOLL [23] the cohesive TSL takes on the form \(t = \partial \psi/\partial \mathbf{u}_c\).

### 3 Representative example

The proposed numerical concept is applied to investigate the dissection of the media of a highly stenotic human iliac artery caused during balloon angioplasty. According to the histological composition and structure, the considered lesion is of type Va, fibroatheroma, according to the classification [24]. This type of stenosis is characterized by prominent new fibrous connective tissue and a lipid core. Collagen is present in the fibrous layer or cap above the lipid core. The 3D geometry is reconstructed from in vitro high-resolution magnetic resonance imaging, where the non-diseased and diseased types of tissues are identified [25].

For simplicity, we neglect residual strains and consider the reconstructed geometry as the (stress-free) reference configuration. Moreover, our model assumes three mechanically different components (see Fig. 1), i.e. Adventitia (A), Media and fibrous Intima (MI), and Lipid Pool (LP). These components are discretized separately, and hence the nodes on their interfaces did not match (Fig. 1). Consequently, the different regions are linked together using standard multi-point constraints.

The goal of the present study is to investigate dis-
Figure 1: Multi-body finite element mesh of a stenotic human iliac artery. Different components are discretized, denoted by Adventitia (A), Media and fibrous Intima (MI), and Lipid Pool (LP). Moreover, the Lumen (L) and the Initial Tear (IT) are indicated.

section of the media secondary to crack propagation in the tissue. Hence, we assume a predefined initial tear in the intima and media (Fig. 1), where the dissection is assumed to propagate. Moreover, the balloon angioplasty intervention is modeled as a contact problem, where standard master-slave concept is applied and the arising contact constraint are enforced using the penalty method.

The material and structural properties determining the bulk response of the artery are taken from [26]. Moreover, (yet unpublished) dissection experiments of the media of human aortas are used to estimate the material properties of the cohesive zone. Based on this investigation, we use $t_0 = 160.0\text{kPa}$, $a = 5.6\text{mm}^{-1}$ and $b = 2.0$ as material parameters.

Figure 2 illustrates the evolution of the maximum principal Cauchy stress in terms of three loading states, where columns (a) and (b) show the stress pattern in $MI$ and $A$, respectively. The deformed mesh of $MI$ clearly indicates that the dissection starts at the initial tear and that the stress concentration at the dissection tip propagates the dissection, which finally causes a cleavage of the arterial wall.

Moreover, once a pronounced dissection is present, the loading mode I is pronounced, and peeling seems to be the underlying dissection mechanism. Once the dissection starts, the main load due to balloon angioplasty is carried by the media, while later on the adventitia contributes to the load carrying mechanisms of the artery. Probably, the most surprising result of the present computation is that the stress level remains moderate, i.e. it does not much exceed above $200.0\text{kPa}$. Hence, we may conclude that the propagating dissection prevents other parts of the artery against damage.

4 Conclusion

Arterial dissections are frequently observed in clinical practice, and, in particular, dissection secondary to crack propagation is typical for balloon angioplasty-induced arterial trauma. The presence of dissections has been shown to be an important contributor of clinical outcome after balloon angioplasty intervention, however, only a view associated biomechanical studies are known in the literature.

The introduced continuum mechanical framework considers strong discontinuities of the displacement field and is entirely formulated within the finite strain domain. It has been assumed that all inelastic effects appear in the cohesive zone and a phenomenological TSL governs its resistance against dissection. The presented formulation is based on the definition of an transversely isotropic cohesive potential in terms of invariants, and a scalar damage parameter characterizes its softening properties. The continuum mechanical model is numerically represented using PUFEM, and a tracking algorithm for 3D non-planar cracks captures the evolution of the dissection.

The dissection of the media due to balloon angioplasty is investigated, where the cohesive properties are estimated from (not yet published) dissection experiments of the media of human aortas. The computation showed that the dissection of the media is primarily of loading mode I with the exception of the crack initialization. Moreover, the maximum principal Cauchy stress remains moderate during the balloon angioplasty intervention and once a pronounced dissection is present, it is redistributed from the media into the adventitia.

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Figure 2: Distribution of the maximum principal Cauchy stress in the arterial wall during balloon angioplasty: (a) Media and fibrous Intima (MI); (b) Adventitia (A).
References


