Chapter 5

ON THE MATHEMATICAL MODELING OF THREE-DIMENSIONAL DELAMINATION PROCESSES OF LAMINATED COMPOSITES

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Abstract The mathematical modeling of 3D delamination failure of laminated composites is discussed. Strong discontinuities are considered in the kinematically framework, which provides the basis for the embedded representation of discontinuities in finite elements. Suitable expressions for a transversely isotropic traction law in form of a displacement-energy function are derived to describe the constitutive response of the interface of laminated composites. Softening phenomena of interfaces are modeled by an isotropic damage law, while the continuous bulk material is modeled as an elastic fiber-reinforced composite. The variational formulation is based on a three-field Hu-Washizu functional which is accompanied with the enhanced assumed strain method. Three different finite element formulations are delineated. A biomechanical example investigates the dissection of the middle layer of a healthy artery, and compares the numerical results of the different finite element formulations obtained from regular and distorted meshes. Soft tissue dissection occurs, for example, during balloon angioplasty, which is a mechanical procedure frequently performed to reduce the severity of atherosclerotic stenoses. Physical and numerical analyses of delamination processes are of pressing scientific and clinical need.

Keywords: Delamination, Dissection, Failure, Discontinuous Finite Element, Cohesive Finite Element, Laminate, Composite, Biomechanics, Artery
1. Introduction

In recent years there has been a significant interest in the discontinuous modeling of strain localization phenomena. To design meaningful constitutive models for capturing localization phenomena and to implement these models in an efficient, fast and numerically accurate way is a major task in science, industrial engineering practice and in the field of biomechanics.

Traditionally, cracks have been represented in discrete or smeared techniques, for an overview see the recent articles Jirásek, 2000 and de Borst, 2001. The discrete approach is more complicated for implementations in a finite element program, and it enforces the direction of crack propagation to be along the element boundaries. Furthermore, problems arise during bandwidth minimization and mesh generation (Lotfi and Shing, 1995). Alternative approaches, in particular for delamination analysis of composites, is the application of (cohesive) interface models, see, for example, Alfano and Crisfield, 2001. The smeared approach treats the cracked material as an equivalent continuum. A standard kinematical representation of highly localized strains, leads, however, to various problems (see Jirásek, 2000) such as overestimation of the stiffness and the strength of structures in shear dominated problems among others (see, for example, Rots, 1988).

In this communication we focus on the mathematical modeling of three-dimensional delamination of composites, in particular, we study the dissection (failure) of an arterial layer, which may be caused through non-physiological loading during, for example, the mechanical procedure of balloon angioplasty. We employ the cohesive crack concept and derive finite elements with embedded strong (displacement) discontinuities which are based on the class of mixed Enhanced Assumed Strain (EAS) method proposed in Simo and Rifai, 1990. The cohesive crack concept combines the strong features of the two traditional approaches (discrete and smeared) suitable to capture cracking.

In Section 2 we introduce kinematical quantities, and characterize strong discontinuities in terms of a jump of the displacement field. We continue with Section 3 by providing an anisotropic model suitable to characterize the mechanical response of fiber-reinforced composites. In addition, we propose a cohesive material model capturing transversely isotropic responses, and assume that all inelastic deformation takes place at the displacement discontinuity. The traction vector is given in terms of the gap displacement, the spatial normal vector and a scalar damage variable – associated spatial tangent moduli are provided. The variational formulation, introduced in Section 4, is based on minimizing a
three-field Hu-Washizu functional, where the compatible displacement, the enhanced strain and the first Piola-Kirchhoff stress tensor are the independent fields. Additionally, we delineate three types of finite element models with embedded discontinuities, the so-called Statically Optimal Symmetric (SOS) formulation, the Kinematically Optimal Symmetric (KOS) formulation, and the Statically and Kinematically Optimal Non-symmetric (SKON) formulations (for an overview see Jirásek, 2000). In Section 5 one representative numerical example compares results obtained from 3D finite element analyses under quasi-static conditions, which are achieved with the three (SOS, KOS, SKON) formulations, and the use of regular and distorted meshes generated with constant-strain tetrahedral elements.

2. Kinematics

In this section we provide the fundamental properties for the (finite) motion and deformation of a continuum including strong discontinuities. We consider a spatially non-continuous motion \( \chi(X,t) \) in the \( n \)-dimensional Euclidean space \( \mathbb{R}^n \). As usual, \( \chi(X,t) \) maps a body \( \mathcal{B} \), occupied by the reference configuration \( \Omega_0 \in \mathbb{R}^n \), into its spatial configuration \( \Omega \in \mathbb{R}^n \), where these configurations are bounded by the surfaces \( \partial \Omega_0 \in \mathbb{R}^{n-1} \) and \( \partial \Omega \in \mathbb{R}^{n-1} \), respectively (see, for example, Holzapfel, 2000).

A strong discontinuity separates the body \( \mathcal{B} \) into the sub-bodies \( \mathcal{B}^i \), \( i = 1, 2 \), occupying the sub-regions \( \Omega^i \), \( i = 1, 2 \), so that \( \Omega = \Omega^1 \cup \Omega^2 \), see Fig. 5.1 (the associated referential sub-regions are \( \Omega_0^i \)). Subsequently, we denote the surfaces occupied by the discontinuity in \( \Omega \) as \( \partial \Omega_i^d \in \mathbb{R}^{n-1} \), \( i = 1, 2 \), with the associated referential surfaces \( \partial \Omega_0^i \in \mathbb{R}^{n-1} \). The remaining boundary surfaces of \( \Omega^i \) are denoted by \( \partial \Omega^i \in \mathbb{R}^{n-1} \) so that \( \partial \Omega = \partial \Omega^1 \cup \partial \Omega^2 \), see Fig. 5.1 (the boundary surfaces of \( \Omega_0 \) read \( \partial \Omega_0 \in \mathbb{R}^{n-1} \)).

As can be seen from Fig. 5.1, motion \( \chi \) maps a point \( X_d \) located at the discontinuity in the reference configuration into two associated points \( x_1^d \) and \( x_2^d \). For characterizing the discontinuity we introduce the outward normal vectors \( \mathbf{N}^i \) and \( \mathbf{n}^i \), \( i = 1, 2 \), to the discontinuities associated with the reference and spatial configurations, respectively. Since a distinction between \( \mathbf{N}^1 \) and \( \mathbf{N}^2 \) is not necessary, in the following we denote the normal vector acting at \( X_d \) just as \( \mathbf{N} \). In addition, we just use \( \partial \Omega_0^d \) for characterizing the referential surface of the discontinuity. Note that for laminated composites normal vector fields within the whole body are defined a priori.
Figure 5.1. Kinematics of a body separated by a displacement discontinuity.

Following Oliver, 1996a, we represent the displacement field \( u(X, t) \) by an additive decomposition into continuous \( u_c \) and discontinuous \( \mathcal{H}_d u_e \) parts. Thus,

\[
u(X, t) = u_c(X, t) + \mathcal{H}_d(X)u_e(X, t) \tag{5.1}\]

holds, where

\[
\mathcal{H}_d(X) = \begin{cases} 0, & X \in \Omega^1_0, \\ 1, & X \in \Omega^2_0 \end{cases} \tag{5.2}\]

denotes the Heaviside function (centered at the discontinuity), which is associated with the reference configuration, and which is independent of time \( t \). Based on definition (5.1) the displacement field \( u_c(X, t) \) is continuous in \( X \) and \( t \), and coincides for all points \( X_d \in \partial \Omega^1_0 \) (equivalent to \( X_d \in \partial \Omega^2_0 \)) on the discontinuity with the displacement jump (or gap displacement), denoted by \( \dot{u}(X_d, t) = x^2_d(X_d, t) - x^1_d(X_d, t) \).

Note that we have limited our considerations to two sub-bodies. The kinematical concept can, however, be easily extended to a finite number of sub-bodies, with non-intersecting discontinuities.

## 2.1 Discontinuous deformation gradient

Here we derive the explicit expression of the discontinuous deformation gradient \( \mathbf{F} \), which is a primary measure of deformation.

We start with the derivation of the material gradient \( \text{Grad} \, u(X, t) \) of the displacement field and use the following (decomposed) representation

\[
\text{Grad} \, u(X, t) = \text{Grad} \, u_c(X, t) + \text{Grad} \, u_e(X, t) + \text{Grad} \, \dot{u}(X, t), \tag{5.3}\]
where $\text{Grad} \, u_c(X,t)$ is the ‘standard’ material gradient of the continuous displacement field $u_c$, while $\overline{\text{Grad}} \, u_e(X,t)$ and $\hat{\text{Grad}} \, u_e$ denote the \textit{bounded} and \textit{unbounded} parts due to the displacement discontinuity $\mathcal{H}_d u_e$. Using eq. (5.1) and the relation $\text{Grad} \mathcal{H}_d(X) = \delta_d(X) N(X,t)$ of the \textit{Heaviside} function, the unbounded contribution reads

$$\hat{\text{Grad}} \, u_e(X,t) = \mathcal{H}_d(X) \text{Grad} \, u_e(X,t),$$

$$\overline{\text{Grad}} \, u_e(X,t) = \delta_d(X) \left[ u_c(X,t) \otimes N(X,t) \right],$$

where

$$\delta_d(X) = \begin{cases} 0, & X \neq X_d, \\ \infty, & X = X_d \end{cases}$$

denotes the \textit{Dirac-delta} functional centered at the discontinuity. Based on eqs. (5.3) and (5.4) the discontinuous deformation gradient is defined to be $F(X,t) = I + \text{Grad} \, u_c(X,t)$, where $I$ denotes the unit tensor. The deformation gradient constitutes the basis for the computation of the right and left Cauchy-Green tensors, i.e. $C = F^T F$ and $b = FF^T$, as used throughout the paper.

### 2.2 Definition of a fictitious discontinuity

As can be seen from Fig 5.1, the opened crack is determined by two surfaces ($\partial \Omega^1_d$ and $\partial \Omega^2_d$) with different outward normal vectors $n^1$ and $n^2$ in the current configuration. In order to provide a description of the (cohesive) discontinuous constitutive model for nonlinear deformation behavior of materials, by following Wells, 2001, we define a \textit{fictitious} discontinuity located in the current configuration. In particular, we introduce the motion $\chi(X_d, t)$ through the deformation gradient $F(X_d, t) = I + \text{Grad} \, u_c + \text{Grad} \, u_e + (\hat{u} \otimes N)/2$ which maps a point $X_d \in \partial \Omega_d$ into a place $x_d$ located on the fictitious discontinuity. The gap displacement at $X_d$ is described by $\hat{u}$. Hence, the normal vector $\vec{n}$ at $x_d$ of the fictitious discontinuity is defined via a weighted push-forward operation of the covariant vector $N$ according to

$$\vec{n} = \frac{N F^{-1}}{|N F^{-1}|}.$$ 

Based on definition (5.6) arbitrary cohesive constitutive relations for transversely isotropic materials can be formulated.

### 3. Material models

In this section we derive and particularize constitutive equations. Since we consider strong (displacement) discontinuities we distinguish
between two types of material models, i.e. continuous and cohesive (discontinuous) material models.

For the continuous constitutive response of the laminated composite we follow Holzapfel et al., 2000a, and use the additive representation of the strain energy

\[ \Psi(C, A_1, A_2) = L(J) + \Psi(C, A_1, A_2), \quad (5.7) \]

where \( J = \det F > 0 \) and \( \overline{C} = J^{-2/3} F^T F \) denotes the Jacobian determinant and the modified right Cauchy-Green tensor, respectively. The first part \( L(J) \) characterizes a mathematically motivated penalty function (with \( L(1) = 0 \)) enforcing the incompressibility constraint. The second part is the energy stored in the material when it is subjected to isochoric elastic deformations. The anisotropic structure of \( \Psi \) is described in terms of the (structural) tensors \( A_1 = a_{01} \otimes a_{01} \) and \( A_2 = a_{02} \otimes a_{02} \), where the unit vector fields \( a_{0i}(X), \) \( |a_{0i}| = 1, i = 1, 2 \), define the preferential fiber directions at point \( X \in \Omega \) (for further details see Holzapfel et al., 2000a).

The cohesive (discontinuous) constitutive model is based on the definition of a displacement-energy function (cohesive potential) with respect to the previous introduced fictitious discontinuity located at the current configuration. We restrict our work to transversely isotropy and introduce

\[ \psi(d, \bar{u}, \bar{n}) = (1 - d)\tilde{\psi}(\bar{u}, \bar{n}), \quad (5.8) \]

where \( \tilde{\psi} \) denotes an effective cohesive potential with respect to the current area \( \overline{\sigma} \) associated with the fictitious discontinuity in the current configuration. In eq. (5.8) \( d \in [0, 1] \) denotes a damage variable describing the state of damage of the interface zone in an isotropic fashion. The standard Coleman-Noll procedure leads to a (constitutive) relation between the (Cauchy) traction vector \( t \), acting at the discontinuity, and the gap displacement \( \bar{u} \). Consequently, the associated tangent moduli \( c_{\bar{u}} \) and \( c_{\bar{n}} \) in spatial descriptions follow from \( t \) by consistent linearization. Thus,

\[ t = (1 - d)\tilde{t}, \quad c_{\bar{u}} = (1 - d)\tilde{c}_{\bar{u}}, \quad c_{\bar{n}} = (1 - d)\tilde{c}_{\bar{n}}, \]

\[ \bar{t} = \frac{\partial \tilde{\psi}}{\partial \bar{u}}, \quad \tilde{c}_{\bar{u}} = \frac{\partial^2 \tilde{\psi}}{\partial \bar{u}^2}, \quad \tilde{c}_{\bar{n}} = \frac{\partial^2 \tilde{\psi}}{\partial \bar{u} \partial \bar{n}}, \quad (5.8) \]

where \( \tilde{t} \) defines the effective Cauchy traction vector, and \( \tilde{c}_{\bar{u}} \) and \( \tilde{c}_{\bar{n}} \) the effective tangent moduli. Note that the tangent modulus \( c_{\bar{u}} \) is symmetric, while \( c_{\bar{n}} \) is non-symmetric, in general.
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The introduced effective cohesive potential \( \tilde{\psi}(\hat{u}, \hat{n}) \) has to obey the standard objectivity requirement, i.e.

\[
\tilde{\psi}(\hat{u}, \hat{n}) = \tilde{\psi}(Q\hat{u}, Q^{-T}\hat{n}),
\]

(5.9)

where \( Q \) is a proper orthogonal tensor, with \( \det Q = +1 \). It describes an arbitrary rigid-body rotation superimposed onto the current configuration. Hence, we assume the effective cohesive potential of the form \( \tilde{\psi}(\hat{u} \otimes \hat{u}, \hat{n} \otimes \hat{n}) \), and introduce the invariant-based representation

\[
\tilde{\psi}(\hat{u} \otimes \hat{u}, \hat{n} \otimes \hat{n}) = \tilde{\psi}(\hat{I}_1, \hat{I}_2, \hat{I}_3, \hat{I}_4, \hat{I}_5),
\]

(5.10)

where the scalar invariants of the set \( \{\hat{u} \otimes \hat{u}, \hat{n} \otimes \hat{n}\} \) of symmetric second-order tensors are defined via (see, for example, Spencer, 1984)

\[
\hat{I}_1 = \text{tr}(\hat{u} \otimes \hat{u}), \quad \hat{I}_2 = \text{tr}(\hat{u} \otimes \hat{u})^2, \quad \hat{I}_3 = \text{tr}(\hat{u} \otimes \hat{u})^3,
\]

\[
\hat{I}_4 = (\hat{u} \otimes \hat{u}) : (\hat{n} \otimes \hat{n}), \quad \hat{I}_5 = (\hat{u} \otimes \hat{u})^2 : (\hat{n} \otimes \hat{n}).
\]

(5.10)

Note, that the introduced structural tensor \( \hat{n} \otimes \hat{n} \) in eq. (5.10) is invariant with respect to the definition of the unit vector \( \hat{n} = -\hat{n} \), which is an essential objectivity requirement.

4. Variational formulation

The variational formulation is based on a three-field Hu-Washizu functional accompanied with the EAS-method, see Simo and Rifai, 1990. According to the EAS-method we assume that

\[
F = I + \text{Grad} u_c + H_e,
\]

(5.11)

where \( u_c \) denotes the compatible displacement field, as introduced in eq. (5.1). The enhanced assumed strain field, denoted by \( H_e \), is a two-point tensor not subject to any interelement continuity requirements.

We consider the three-field Hu-Washizu functional to be of the form \( \Pi(u_c, H_e, P) \), where \( u_c, H_e \) and \( P \) denote the three independent fields, the compatible displacement, the enhanced strain and the first Piola-Kirchhoff stress tensor, respectively. Note that, by satisfying angular momentum in an arbitrary point \( X \), i.e. \( PF^T(u_c, H_e) = F(u_c, H_e)P^T \), a constraint is enforced onto the (otherwise independent) three fields. Consequently, three stationarity conditions (variational statements) may
be derived according to (Simo and Armero, 1992)

\[ \int_{\Omega_0} \text{Grad} \delta u_c : \left( 2F \frac{\partial \Psi(C)}{\partial C} \right) dV - \delta \Pi_{\text{ext}} \delta u = 0, \]

\[ \int_{\Omega_0} \delta H_e : \left[ -P + \left( 2F \frac{\partial \Psi(C)}{\partial C} \right) \right] dV = 0, \]

\[ \int_{\Omega_0} \delta P : H_e dV = 0, \]

(5.12)

where \( \delta u_c, \delta P \) and \( \delta H_e \) are admissible variations of \( u_c, P \) and \( H_e \), respectively. Note that in eq. (5.12) appears no variation of the (total) gradient \( H \) rather then the enhanced part \( H_e \). In addition, \( \delta \Pi_{\text{ext}}(\delta u) \) and \( dV \) denote the variation of the potential energy of the external loading and the volume element in the reference configuration, respectively.

Similar to the introduced kinematics we assume that the enhanced strain field \( H_e \) (and the associated first variation \( \delta H_e \)) can be represented by the additive decomposition \( H_e = H + \hat{H} \) into a bounded contribution \( H \) and an unbounded contribution \( \hat{H} \). In order to satisfy the orthogonality condition (5.12) (in a weak sense) we employ the Galerkin method, which leads to a two-field problem, where the independent stress field \( P \) disappears from the variational equation (5.12), see Simo and Rifai, 1990. This approach leads to the SOS formulation, which is closely related to smeared approaches, which, therefore, show the typical stress-locking phenomena (for an overview see Jirásek, 2000).

In order to overcome the locking phenomena, Dvorkin et al., 1990 (with generalizations introduced by Simo and Oliver, 1994, Oliver, 1996a and Oliver, 1996b) proposed a non-symmetric formulation called the SKON formulation. In this approach the kinematics of strong discontinuities (as introduced in the Section 2) is fulfilled properly, however this approach uses the Petrov-Galerkin method in order to satisfy eq. (5.12). Furthermore, the stress field \( P \) in eq. (5.12) does not disappear, although the SKON formulation deals with a two-field approach. Hence, this approach is not consistent with the variational formulation since it is assumed that \( H_e \) and \( \delta H_e \) are elements of the same space, which is not the case for the Petrov-Galerkin method.

Finally the KOS formulation (see Lotfi and Shing, 1995), which is based on the Galerkin method, describes the kinematics satisfactorily but it does not satisfy the orthogonality condition (5.12).
Representative numerical example

This section has the goal to investigate the efficiency of the proposed framework by comparing the numerical results obtained from the three classes of models, i.e. SOS, KOS and SKON. The introduced cohesive finite element formulation has been implemented into the multi-purpose finite element analysis program FEAP (Taylor, 2000), and constant-strain tetrahedral elements are used.

5.1 Dissection of the middle layer of an artery

In the example we perform a finite element analysis of the middle layer (the media) of an artery considered to be delaminated (dissected). The media is an important load-carrying layer in a healthy artery, and is roughly composed by a varying number of well-defined concentrically fiber-reinforced layers (lamellae). For a more detailed exposition see Holzapfel et al., 2000a, Section 2, and Holzapfel, 2002 with more references therein. Balloon dilations of arteries, frequently performed to reduce the severity of atherosclerotic stenoses, often leads to tissue dissections, which to analyze computationally is of pressing scientific and clinical need. The media is modeled as a 3D problem, where regular and distorted meshes are used for reason of comparison.

The reference geometry of the media, considered as a fiber-reinforced strip, is shown in Fig. 5.2, where \( L = 10.0 \text{ (mm)} \) and \( A = 1.0 \text{ (mm)} \) are the dimensions. The bottom face of the strip is fixed. The collagen fibers and the interface zones are schematically shown in Fig. 5.2. We consider a displacement-driven problem, whereby the displacement of the rigid component, as indicated in Fig. 5.2, is prescribed along the \( x_2\)-
direction. The rigid body transmits the load into the strip, which leads to a desired crack initialization in the middle of the specimen. During additional loading the stress state determines the crack propagation. The calculation is performed under 3D conditions, where symmetry is assumed with respect to the plane \( x_3 = 0 \).

The continuous model is given by the (strictly convex) penalty function \( L(J) = \kappa (J - 1)^2 / 2 \), where the penalty parameter \( \kappa \) is chosen to be 
\[
100.0 \text{ (mN/mm}^2) = 100.0 \text{ (kPa)}.
\]
The material parameters for the (isotropic) matrix material and the fibers are adopted from Holzapfel et al., 2000b. The angle between the two fiber directions in the reference configuration is given by \( \gamma = 14.0^\circ \).

In view of missing experimental data for the interface of the lamellar units we use a linear elastic particularization of eq. (5.8), and propose
\[
\psi(d, \hat{I}_1, \hat{I}_4) = (1 - d) \left[ \frac{c_n}{2} \hat{I}_4 + \frac{c_t}{2} (\hat{I}_1 - \hat{I}_4) \right],
\]
with
\[
d = 1 - \exp \left( -a \hat{u}_H^b \right),
\]
where \( c_n \) and \( c_t \) are material parameters characterizing the normal and tangential responses, respectively. The evolution of the damage at the interface is described by the dimensionless material parameters \( a \) and \( b \), while \( \hat{u}_H(X_d, t) = \max \| \hat{u}(X_d, s) \|_{s=0} \) is the magnitude of the maximum gap displacement of the whole deformation path of a particular point \( X_d \) at the interface at time \( t \). Hence, using the particularization (5.13) and eqs. (5.7) and (5.8) it is straightforward to derive the Cauchy traction vector \( \mathbf{t} \) and the associated tangent moduli \( c_\mathbf{u} \) and \( c_\pi \).

For the numerical example we used the material parameters
\[
c_\pi = 1000.0 \text{ (mN/mm}^2) \quad \text{and} \quad c_t = c_\pi / 2
\]
for the normal and tangential responses, respectively. In addition, isotropic damage is specified by the parameters \( a = 8.0 \text{ (mm}^{-1}) \) and \( b = 0.5 \). This set of parameters is associated with about 8.5 (mN/mm\(^2\)) \( \equiv 8.5 \text{ (kPa)} \) cohesive strength for mode I and about 4.25 (kPa) for mode failure II and III. In order to avoid penetration of the cohesive zone a penalty constraint is introduced, where the penalty (normal) stiffness of 1000.0 (N/mm\(^3\)) was taken into account. The fully 3D problem is computed with regular and distorted meshes of 20580, 7500 and 1620 elements.

In Fig. 5.3 the load-displacement responses for the 3D computations are shown. On the regular meshes all three finite element formulations lead to the same result and on the distorted meshes the SOS formulation
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leads to the typically stress locking phenomena, see Fig. 5.3(a). The KOS formulation leads even on the distorted mesh to satisfying results, see Fig. 5.3(b). With the SKON formulation and for distorted meshes we were not able to compute the whole dissection process with the fixed load step of 0.1 (mm), see Fig. 5.3(c).

In Fig. 5.4 the maximum principal Cauchy stresses, in (mN/mm$^3$), are plotted onto the deformed configurations during the dissection process. Distorted meshes of 1620 and 7500 tetrahedral elements are employed and the KOS formulation is used. Note the stress concentration around the delamination front, and the good qualitatively agreement for both calculations. A second region of stress concentration can be observed around the location where the load is transmitted into the strip. More detailed investigations have shown that the maximum principal stresses in this region are aligned with the interface such that no further delamination took place.

6. Conclusion

Mathematical modeling of three-dimensional delamination processes of laminated composites was presented. In particular, the delamination (dissection) of the middle layer of a healthy artery — regarded as a highly deformable composite structure — was studied in more detail. Different EAS-based finite element formulations with embedded strong discontinuities are compared and discussed.

Dependent on the interpolation of the enhanced assumed strain field $H_e$ (and its associated admissible variation) one can set up three classes of finite element models, known as statically optimal symmetric (SOS), kinematically optimal non-symmetric (SKON), and kinematically optimal symmetric (KOS). While the SOS formulation satisfies a three-field Hu-Washizu variational principle, it shows pronounced stress-locking phenomena on distorted meshes. Both the SKON and KOS formulations overcome stress-locking, however, they are not consistent with the variational principle. On the basis of the delamination (dissection) of an arterial layer, modeled as a 3D problem with constant-strain tetrahedral elements, regular and distorted meshes are used to compare the associated load-displacement responses. As expected, stress locking phenomena were seen for the SOS formulation. Only the KOS formulation has led to meaningful results on distorted meshes. It was not possible to achieve similar results on distorted meshes with the SKON formulation. A comparative study of the maximum principal Cauchy stresses for distorted meshes of 1620 and 7500 tetrahedral elements have given qualitative good agreement.
Figure 5.3. 3D dissection analysis of the middle layer of an artery. Load-displacement response for the (a) SOS, (b) KOS and (c) SKON formulations. Regular and distorted meshes using 20580, 7500 and 1620 elements are used. Note the stress locking effects accompanied with the SOS formulation for distorted meshes. With the SKON formulation and for distorted meshes no meaningful 3D results could be achieved with the fixed load step of 0.1 (mm).
Figure 5.4. Maximum principal stresses, in (mN/mm²), are plotted onto the deformed configurations during the dissection process. The 3D computation is based on the KOS formulation. Slightly distorted meshes with 1620 and 7500 tetrahedral elements were used.

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