A hyperelastic biphasic fibre-reinforced model of articular cartilage considering distributed collagen fibre orientations: continuum basis, computational aspects and applications

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Cartilage is a multi-phase material composed of fluid and electrolytes (68–85% by wet weight), proteoglycans (5–10% by wet weight), chondrocytes, collagen fibres and other glycoproteins. The solid phase constitutes an isotropic proteoglycan gel and a fibre network of predominantly type II collagen, which provides tensile strength and mechanical stiffness. The same two components control diffusion of the fluid phase, e.g. as visualised by diffusion tensor MRI: (i) the proteoglycan gel (giving a baseline isotropic diffusivity) and (ii) the highly anisotropic collagenous fibre network. We propose a new constitutive model and finite element implementation that focus on the essential load-bearing morphology: an incompressible, poroelastic solid matrix reinforced by an inhomogeneous, dispersed fibre fabric, which is saturated with an incompressible fluid residing in strain-dependent pores of the collagen–proteoglycan solid matrix. The inhomogeneous, dispersed fibre fabric of the solid further influences the fluid permeability, as well as an intrafibrillar portion that cannot be ‘squeezed out’ from the tissue. Using representative numerical examples on the mechanical response of cartilage, we reproduce several features that have been demonstrated experimentally in the cartilage mechanics literature.

Keywords: cartilage; constitutive modelling; finite element simulation; collagen fibre; biphasic; homogenisation

1. Introduction

Cartilage, a multi-phase material composed of fluid, electrolytes, chondrocytes, collagen fibres, proteoglycans and other glycoproteins (cf. Mow et al. 2005; Athanasiou et al. 2010), contains a fibre network of predominantly type II collagen, which provides tensile strength and stiffness to the composite tissue. Figure 1 schematically shows the layered structural organisation of type II collagen fibre. An overview on the structure and function of collagen in articular cartilage can be found in Responde et al. (2007).

Finite element (FE) simulation is a well-accepted means to gain insight on the functional relationships between structure and mechanical properties within articular cartilage. Three-dimensional (3D) FE modelling of articular cartilage has been accomplished using single-phase, biphasic and multi-phasic models. Note that while we include some axisymmetric modelling approaches in the discussion, such formulations do not allow true 3D collagen fibre orientations to be implemented and this shortcoming might affect solutions in non-intuitive ways (Li et al. 2009). Li et al. (2001) investigated the variability of a 3D FE model constructed from MR images of a human knee joint using the stress results generated under axial compressive loads. This work employed a homogeneous, linear elastic single-phase model, and varied the effective material properties to simulate ‘short-term’ versus ‘long-term’ responses (Young’s modulus, $E = 5$–10 MPa and Poisson’s ratio, $\nu = 0.46$), to predict the short-term cartilage response to compressive loading, and thus to study the effects of meniscal tears and meniscectomies on cartilage stresses and progression of degeneration.

Incompressible, single-phase models are appropriate for modelling articular cartilage undergoing relatively rapid loads in short-time durations due to the equivalence of the short-time biphasic and incompressible elastic responses for arbitrary cartilage deformations (Athesian et al. 2007). In this vein, Anderson et al. (2008, 2010) used a neo-Hookean material (shear modulus, $\mu = 2.68–13.6$ MPa), and varied the effective Poisson’s ratio from 0.495 to 0.452, to model articular cartilage in the human hip joint. They developed and validated a subject-specific FE model of hip joint contact mechanics under physiological loading to predict contact pressures accurately, not stresses within the cartilage volume. Pierce et al. (2009) proposed an isochoric, finite-strain viscoelastic fibre-reinforced constitutive model, which separated the solid matrix and fibre fabric viscous contributions. The model was capable of reproducing the...
deformation behaviour of human articular cartilage in the physiological loading domain, as demonstrated by the good agreement between the experiment and numerical results.

Constitutive models describing multi-phasic materials are generally based on mixture theory (Whitaker 1977; Hassanizadeh and Gray 1979a, 1979b, 1980; Mow et al. 1989; Lai et al. 1991), the theory of poromechanics (Abousleiman et al. 1996; Chen and Detournay 1998; Coussy 2004) and the theory of porous media (Bowen 1980, 1982; Ehlers 1989, 1993; de Boer 2000). Biphasic cartilage models, including both solid and fluid contributions of the tissue response, are naturally more realistic, but also more computationally expensive. Li et al. (2000) developed a 2D axisymmetric FE modelling approach, which included a linear elastic isotropic proteoglycan matrix, collagen fibrils (modelled as linear springs) and fluid. By employing depth-dependent material properties estimated from the literature, they demonstrated the necessity of inhomogeneous modelling approaches for simulating the mechanical behaviour of cartilage. Korhonen et al. (2003) modelled the effects of proteoglycan depletion or collagen degradation using a fibril-reinforced poroelastic FE model in 2D axisymmetry by reducing Young’s modulus of either the matrix or the fibril network, respectively. Results indicate that their model could predict alterations in the stress-relaxation behaviour of cartilage after enzymatic treatments. Han et al. (2005) studied articular surface contact in normal and misaligned patellofemoral joints in cats using an incompressible, linear elastic solid phase and an incompressible, non-viscous fluid phase. Manda et al. (2011) employed a similar model (fully saturated, linear elastic porous solid) to study the effects of metal implants on the stress distributions and deformations in articular cartilage of knee joints in sheep. Li et al. (2009) modelled cartilage as a two-phase mixture undergoing small deformations, using a linear elastic matrix with a nonlinear viscoelastic fibril reinforcement and a hydraulic permeability defined by the commercial FE software ABAQUS (Simulia Corp., Providence, RI, USA). They observed significant fibre orientation dependence in the displacement, fluid pressure and velocity prior to equilibrium. Gu and Li (2011) used this constitutive model and an anatomically correct whole joint geometry (constructed from MR images), with realistic fibre orientations for the surface zone of the cartilage, to study the significance of fluid pressurisation and fibre reinforcement under small deformations. Their results indicate the necessity of including fluid pressure and anisotropic fibril reinforcement in cartilage constitutive models for a more accurate understanding of the mechanics of the knee. Mononen et al. (2012) studied the effect of collagen fibre orientation and superficial collagen fibrillation on cartilage stresses and strains within a patient-specific knee geometry using a porohyperelastic (non-fibrillar) matrix and a viscoelastic fibrillar matrix. Their results demonstrate that the organisation of collagen fibres has important ramifications for the weight-bearing function of articular cartilage.

Three-dimensional FE analysis of articular cartilage using biphasic swelling or three-phase (solid, fluid and ion concentration) or multi-phase models generally adds osmotic pressure (swelling), which results primarily from Donnan and entropic effects (see review in Chahine et al. 2005), to further increase the complexity of the model. van Loon et al. (2003) presented a quadriphasic mixture model (homogeneous, isotropic, materially incompressible and charged solid; fluid; monovalent cations and monovalent anions) intended to apply to biological tissues. They employed a 3D FE implementation of the model to qualitatively simulate the swelling of a cylindrical hydrogel sample. Wilson et al. (2005) developed a fibril-reinforced poroviscoelastic swelling model for articular cartilage which included both primary and secondary (smaller) collagen fibrils, and swelling properties due to the fixed charge densities of the proteoglycans. They showed that 2D axisymmetric FE simulations can simultaneously predict reaction forces during swelling, confined compression, indentation and unconfined compression (including lateral deformation during unconfined compression). Chen et al. (2006) completed a nonlinear 3D FE analysis of an articular
cartilage tissue free-swelling problem to reproduce the experimentally determined curling behaviour of in vitro cartilage strips submerged in solution baths of various concentrations. Their model, while isotropic, accounts for deformation-dependent permeability and non-uniform distributions of fixed charge density and solidity. Ateshian et al. (2009) modelled the solid matrix of cartilage with a continuous fibre angular distribution (where fibres can only sustain tension) swelled by the osmotic pressure of a proteoglycan ground matrix. Their constitutive model for cartilage can predict a number of observed phenomena when flow-dependent and flow-independent viscoelastic effects have subsided. To model charged, hydrated tissues, Ehlers et al. (2009) applied the theory of porous media in combination with large-strain, Ogden-type material laws describing anisotropic and viscoelastic behaviours of the solid matrix and osmotic effects based on the simplifying assumption of Lanir. While the modelling framework, demonstrated in 3D FEIs, is applied to human intervertebral disc as a representative test case, it could also be applied to articular cartilage.

Many studies have pursued destructive (possibly with staining) and non-destructive experimental methods to characterise the fibre orientation and density, e.g. both multiphoton microscopy (Zipfel et al. 2003; Lilledahl et al. 2011) and diffusion tensor MRI (DT-MRI; Filidoro et al. 2005; Meder et al. 2006; deVisser et al. 2008a; Pierce et al. 2010). An overview of current imaging techniques for articular cartilage can be found in Potter et al. (2009). DT-MRI can be employed to measure quantities describing cartilage microstructure (i.e. the collagen fibre fabric) on micron length scales due to the influence of this ‘fabric’ on the fluid transport properties of the tissue. The solid phase of cartilage constitutes an isotropic proteoglycan gel and a fibre network of predominantly type II collagen. Diffusion of the fluid phase is principally controlled by the same two components: (i) the proteoglycan gel that gives a baseline isotropic diffusivity and (ii) the highly anisotropic collagenous fibre network. Collagen fibre bundles restrict diffusion, leading to a lower diffusivity normal to the fibres and a correspondingly higher diffusivity in the direction parallel to the local fibre orientation (Meder et al. 2006; deVisser et al. 2008a).

Motivated by these observations, which have not yet been captured with 3D modelling, we propose a new constitutive model and FE implementation that focus on the essential load-bearing morphology: an incompressible, poroelastic solid matrix, reinforced by an inhomogeneous, dispersed fibre fabric, saturated with an incompressible fluid at constant electrolytic conditions residing in strain-dependent pores of the collagen–proteoglycan solid matrix. Furthermore, the inhomogeneous, dispersed fibre fabric of the solid influences the fluid permeability, as well as an intrafibrillar portion that cannot be ‘squeezed out’ from the tissue. The material parameters are structurally motivated and have a direct physical interpretation.

To demonstrate the utility of our proposed modelling approach, we reproduce several tests that have been demonstrated experimentally in the cartilage mechanics literature, e.g. solid and fluid anisotropy, tension–compression nonlinearity, creep displacement with significant fluid pressure load support, variable instantaneous and equilibrium Poisson’s ratios, as well as inhomogeneous, depth-dependent strain. Huang et al. (2001) noted that, ‘While a guiding principle of constitutive modeling of biological tissues is to adopt the simplest possible formulation that can describe experimental data, it is becoming increasingly clear that the complexity of articular cartilage mechanics requires more elaborate models than those already presented in the literature’. While many researches have contributed significantly to the field of 3D FE modelling of articular cartilage, the statement is still true today. As such, we place features of the proposed model in context and discuss advantages and limitations of this modelling approach.

2. Mathematical model

We describe articular cartilage as a biphasic continuum \( \varphi = \varphi^S + \varphi^F \), which consists of the porous solid phase \( \varphi^S \) filled with the fluid phase (intrinsic water) \( \varphi^F \). The solid phase represents the porous tissue structure with an isotropic and statistically regular pore distribution. Moreover, the solid phase consists of an incompressible matrix with an imbedded reinforcing collagen fibre fabric. The fluid phase represents the pore water, where the fluid flow is influenced by the outer pressure gradient. To describe this biphasic system, we use the theory of porous media as a superordinate framework which offers the possibility of simulating fluid saturated porous structures within a coupled, superimposed, continuum mechanical framework (see Bowen 1980, 1982; Ehlers 1989, 1993; de Boer 2000). The mixture model leads to a numerical solution of the coupled solid–fluid behaviour in the framework of a FE formulation.

2.1. Theory of porous media: fluid flow, mixture theory and volume fractions

We now represent the microscopic tissue structure with a statistical distribution of the constituents over a representative elementary volume in such a way that we are allowed to describe the distribution of the constituents by an average volume fraction \( n^\alpha \). The volume fractions \( n^\alpha \) refer the volume elements \( dV^\alpha \) of the individual constituents \( \varphi^\alpha \) to the bulk volume element \( dV \)

\[
\sum_{\alpha=1}^{k} n^\alpha (x,t) = \sum_{\alpha=1}^{k} \rho^\alpha = 1, \quad \alpha \in \{S,F\},
\]

\[
\frac{dV^\alpha}{dV} = \frac{\rho^\alpha}{\rho^S + \rho^F},
\]

where \( \rho^\alpha \) is the mass density of each component.
where $\mathbf{x}$ is the position vector of the spatial point $x$, $t$ is the time and $S$ and $F$ denote the solid and fluid, respectively. From (1), we develop a homogenised model with superimposed continua. The volume fractions $n^a$ in (1), satisfy the saturation condition (1)$_2$ for $k$ constituents $\varphi^a$. Moreover, the partial density $\rho^a = n^a \rho^{a \text{R}}$ of the constituent $\varphi^a$ is related to the real density of the materials $\rho^{a \text{R}}$ involved via the volume fractions $n^a$, see (1)$_2$. Due to the volume fraction concept, all geometric and physical quantities, such as motion, deformation and stress, are defined in the total control space. Thus, they can be interpreted as the statistical average values of the real quantities.

We treat the saturated porous solid as an immiscible mixture of constituents $\varphi^a$ with particles $X_a$, where at any time $t$ each spatial point $x_S$ of the current solid placement is simultaneously occupied by fluid particles $x_F$. These particles proceed from different reference positions $X_a$ at time $t = t_0$ and thus each constituent is assigned its own independent motion function. Furthermore, the Jacobian is defined as $J_a = \det\mathbf{F}_a$, where $\mathbf{F}_a = \partial \mathbf{x}_a / \partial \mathbf{x}$ is the deformation gradient of the constituent $\varphi^a$. During the deformation process $\mathbf{F}_a$ is restricted to $\det \mathbf{F}_a > 0$.

For scalar fields depending on $\mathbf{x}$ and $t$, we define the material time derivatives as $(\cdot)_t = \partial (\cdot) / \partial t + [\text{grad}(\cdot)] \mathbf{v}_a$, with $\text{grad}(\cdot) = \partial (\cdot) / \partial \mathbf{x}$. In order to use a material objective measure of the fluid velocity with respect to the solid velocity, we introduce the seepage velocity $\mathbf{w}_{FS}$, which describes the difference in velocity between the fluid phase $\mathbf{x}_F$ and the solid phase $\mathbf{x}_S$ as $\mathbf{w}_{FS} = \mathbf{x}_F - \mathbf{x}_S$. In connection with the fluid volume fraction this leads to the definition of the filtration velocity $n^F \mathbf{w}_{FS}$. For an extended explanation of the kinematics of porous media, see de Boer (2000) or Ehlers (2002).

### 2.2. Assumptions

Our constitutive model is based on four assumptions. First, we assume that the cartilage tissue consists of two phases: a solid skeleton $\varphi^S$ saturated by a pore fluid $\varphi^F$. Thus, we assume that the total ion concentration of the tissue remains constant (i.e. constant electrolytic conditions such that the Donnan osmotic pressure does not change). In practice, the magnitude of the swelling strain is relatively small in comparison to the tissue strains under physiologic loading conditions (Soltz and Ateshian 2000). Hence, we are justified in approximating the response with a biphasic model. In biphasic theory, the Donnan osmotic pressure is lumped with the stiffness of the solid matrix (Soltz and Ateshian 1998).

Second, we assume that both phases are individually incompressible ($(\rho^S)^\prime_0 = 0,$ and $(\rho^F)^\prime_0 = 0$). The mixture of collagen type II and proteoglycans in articular cartilage shows negligible volume change during hydrostatic pressurisation (see, e.g. Bachrach et al. 1998). The volumetric deformation of the mixture body results from the change in pore space, namely a change in the volume fraction $n^a$, which leads to a macroscopic volumetric deformation. This idealisation is valid for most biological tissues since the volume change in the real material is negligible in contrast to the pore deformation (see, e.g. Humphrey 2002). In a cartilage tissue, a volume deformation due to a physiological hydrostatic pressure is also not observed experimentally (Jurvelin et al. 1997; Bachrach et al. 1998; Soltz and Ateshian 1998; Wong et al. 2000; Park et al. 2003; Huang et al. 2005).

Since no mass or volume exchanges occur during deformation, our third assumption is that mass exchanges between the solid and the fluid constituents can be neglected (immiscibility). Finally, we assume that the dynamic effects are negligible (quasi-static), so we restrict both phases to exclude inertial terms in the dynamics Equations ($x'_a = 0$).

### 2.3. Field equations

Based on the above (four) assumptions, we generate a quasi-static, two-phase model with individually incompressible phases under isothermal conditions without mass exchanges. In order to calculate the unknown quantities, i.e. the motion of the solid $\mathbf{u}_S$ and the fluid velocity $\mathbf{v}_F$, we require independent field equations. Under the modelling assumptions, the set of field equations contains the volume and momentum balances as

$$
(n^S)_t + n^F \text{div} \mathbf{v}_F = 0, \quad \text{div} \mathbf{a}^S + \rho^S \mathbf{b} + \mathbf{p}^S = \mathbf{0},
$$

where $\text{div}(\cdot)$ denotes the spatial divergence operator, $\mathbf{a}^a$ is the partial Cauchy stress tensor, $\mathbf{b}$ is the body force per unit mass and $\mathbf{p}^a$ describes the interaction forces of the constituents $\varphi^a$, which are restricted to $\mathbf{p}^S + \mathbf{p}^F = \mathbf{0}$. The arguments for the Helmholtz free-energy functions are postulated as $\Psi^S = \Psi^S(C_S)$ and $\Psi^F = \Psi^F(C_F, \ldots, \cdot)$, where $C_S = F^T_S F_S$ represents the right Cauchy–Green tensor. For simplicity, and as usual in incompressible fluid mechanics, we assume that the Helmholtz free energy of the fluid phase does not depend on the deformation gradient $\mathbf{F}_F$ when viscous properties are negligible (based on dimensional arguments; see, e.g. Hassanizadeh and Gray 1990 or Markert 2007). Thus, $\Psi^F$ does not depend on any process variable.

In the case of a biphasic model with two material incompressible constituents, a macroscopic volumetric deformation can only be achieved through a change in the volume fraction. In order to account for the saturation constraint, we constrain the material time derivative of the saturation condition (1)$_2$, following the motion of the solid phase as follows:

$$
(n^S)_t + (n^F)_t = 0.
$$

By evaluating the entropy inequality of the mixture (see Ricken and Bluhm 2010), it can be shown that the motions of
both the solid and fluid phases are connected by the interaction forces \( \mathbf{p}^F = -\mathbf{p}^S \)
with
\[
\mathbf{p}^F = \rho \text{grad} n^F - \mathbf{R}_c \mathbf{w}_{FS},
\]
where \( \rho \) denotes the fluid pressure and \( \mathbf{R}_c \) denotes a positive definite material parameter tensor representing the intrinsic hydraulic resistance of the cartilage solid matrix (detailed in Section 2.5).

### 2.4. Solid and fluid stresses

We propose the constitutive relations for the partial Cauchy stress tensors as
\[
\mathbf{\sigma}^S = -n^S \rho \mathbf{I} + 2\rho S F^S \frac{\partial \Psi^S}{\partial C_S} F^S = -n^S \rho \mathbf{I} + \mathbf{\sigma}^S_E,
\]
\[
\mathbf{\sigma}^F = -n^F \rho \mathbf{I},
\]
where \( \mathbf{\sigma}^S_E \) is the effective Cauchy stress tensor (see, e.g. Bishop 1959 or Skempton 1960).

Bachrach et al. (1998) argue that anisotropy arises primarily from the collagen fibre fabric, so that the remaining matrix material is best treated as isotropic. In the case of (statistically) homogeneously distributed pores, the isotropy of the solid material is also not affected by the pores. Furthermore, for the low loading domain, the (wavy) collagen fibres of soft connective tissues are not active (they do not store strain energy). An anisotropic strain response can be observed in the high loading domain due to the collagen fibre reinforcement. To model the resulting (locally) transversely isotropic stress behaviour, we use the concept of integrity bases, allowing a coordinate-invariant formulation (see, e.g. Spencer 1971; Betten 1987; Boehler 1987; Zheng and Spencer (1993a, 1993b). This leads to an additive decomposition of the superimposed solid Helmholtz free-energy function \( \Psi^S \) into an isotropic matrix part \( \Psi^S_{IM} \) and a transversely isotropic (fibre fabric) part \( \Psi^S_{FF} \), i.e.
\[
\Psi^S = \Psi^S_{IM}(J_S, I_1) + \Psi^S_{FF}(I_1, I_4),
\]
where \( I_1 := \text{tr} C_S, I_4 := C_S : M_0, \) with \( M_0 = a_0 \otimes a_0 \) a (Lagrangian) structural tensor, and the local principal direction of the collagen fibres is characterised by the (reference) direction vector \( a_0 \), with \( |a_0| = 1 \) (such that anisotropy arises only through the modified invariant \( I_4 \)). Note that \( I_4 \) has a clear physical interpretation as it is the square of the stretch of the collagen fibre fabric in the direction \( a_0 \). Finally, the isotropic matrix \( \Psi^S_{IM} \) and fibre fabric \( \Psi^S_{FF} \) contributions to the function \( \Psi^S \) must be particularised in a manner capable of reproducing the experimentally observed cartilage response.

In the case of volumetric compression, a compaction point must be introduced defining the state where all fluid is pressed out of the tissue and all pores are closed, so that no further compression can occur. Several strain energy functions have previously included a compaction point, e.g. in the context of general multi-phase models (e.g. Ehlers and Eipper 1997, 1999), models of soft biological tissues (e.g. cardiac tissue; Chapelle et al. 2010) and models of cartilage specifically (e.g. Wilson et al. 2007; Federico and Grillo 2012). Since in biological tissues, neither the intracellular nor the interstitial fluid can be pressed out fully, the compaction point cannot be achieved physiologically by mechanical loading only. However, due to the theoretical existence of the compaction point, we modify the nonlinear relationship between stress and volumetric strain. We use the Helmholtz free-energy function based on the formulation in Bluhm (2002) who extends an energy function based on Simo and Pister (1984) to describe the compaction effect as
\[
\Psi^S_{IM}(J_S, I_1) = \frac{1}{\rho_{IS}} [U(J_S) + \frac{1}{2} \mu^S (I_1 - 3)],
\]
where
\[
U(J_S) = \lambda^S \left[ \left( \frac{1}{J_S} \right)^2 + \xi^S \right] - \mu^S \log J_S
\]
and where the abbreviations
\[
\lambda_{cp}^S = \lambda^S \left[ 1 + J_S^c \left( \frac{1 + (J_S^c)^2}{1 - J_S^c} \right) \right]^{-1},
\]
\[
\xi^S = J_S^c \log J_S + \frac{1 - J_S^c}{J_S^c - 2} \left[ \log \left( \frac{J_S^c - J_S}{J_S^c - 1} \right) - \log \left( 1 - J_S^c \right) \right]
\]
have been used. In (7)–(9), \( \mu^S \) is Lamé’s second parameter (a stress-like material parameter, corresponding to the shear modulus of the underlying matrix material in the reference configuration), \( \lambda^S \) is Lamé’s first parameter (a stress-like material parameter, which in the case of isochoric deformation of the solid matrix degenerates to a non-physical, (positive) penalty parameter used to enforce incompressibility) and \( J_{cp}^f = n_{0S}^f \) defines the point of compaction for an incompressible solid. In cartilage, it is generally not possible to ‘squeeze out’ all of the fluid present in the tissue, so this condition is modified to \( n_{0S}^f \leq J_{cp}^f \leq 1 \). By employing this formulation, it is possible to account for the influence of the mixture incompressibility during the deformation of the pores with respect to the true incompressible solid content.

We capture the anisotropic and nonlinear response of the dispersed collagen fibre fabric phenomenologically by a strain-energy function extended to consider the dispersion of the collagen fibre orientation as (Holzapfel et al. 2005a,
In order to determine the filtration velocity \( n^F w_{FS} \), we rearrange (11) into

\[
\frac{\langle n^F w_{FS} \rangle}{R_F} = (n^F)^2 \frac{R_F^{-1}}{-\nabla p + \rho^F b}. \tag{12}
\]

Thus informed, and due to the fact that the motions of both solid and fluid are connected by the interaction forces \( \mathbf{p}^F = -\mathbf{n}^F \), and considering the thermodynamic restriction of (4) (see, e.g. Ricken and Bluhm 2009, 2010), we propose an anisotropic intrinsic permeability of the cartilage solid matrix \( K_F \) in the form (similar to Ricken et al. 2010)

\[
(n^F)^2 R_F^{-1} = K_F = \left( \frac{n^F}{1 - n_{OS}^F} \right) k_{OS} \left[ (1 - \rho) I + \rho \frac{\mathbf{M}}{I_s} \right],
\tag{13}
\]

where the permeability depends on the deformation and is characterised using the initial Darcy permeability \( k_{OS} \) (m\(^2\)/Ns) and \( \rho \) (a dimensionless material parameter controlling the general isotropic deformation dependence of the permeability; also see, e.g. Eipper 1998). Inclusion of the volume fraction \( n^F \) relates to the change in permeability caused by the change in pore space, where \( n_{OS}^F \) denotes the reference solid volume fraction. The (spatial) structural tensor \( \mathbf{M} \) is defined \( \mathbf{M} = \mathbf{a}_s \otimes \mathbf{a}_s = F_{Sa} a_0 \otimes F_{Sa} a_0 = F_s M_a F_a^T \). Equation (10) employs both the direction vector \( \mathbf{a}_s \) and the parameter \( \rho \in [0,1] \) to specify the (local) principal direction and dispersion in the collagen fibre fabric. Similarly, \( \rho \) is used here to define the range of permeabilities resulting from ideal alignment of collagen fibres (0% weight on the isotropic distribution, \( \rho = 1 \)) to an isotropic distribution of the collagen fibre fabric (100% weight on the isotropic distribution, \( \rho = 0 \)).

### 3. Numerical implementation

The numerical implementation is very similar to that detailed in Ricken et al. (2010), which we condense and review here for completeness and differences in notation. We consider the solid body \( B = B_s \) with the material surface \( \partial B = \partial B_s \), where the solid body \( B_s \) is a subset of the fluid body \( B_f \) with \( B_f \subset B_f \). Note that for the fluid phase, \( \partial B_f \) is a non-material surface, i.e. the material surface \( \partial B_f \) of the fluid is always outside the solid body (see Figure 2(a)). Under consideration of \( \partial B_s \subset \partial B_u \cup \partial B_w \), with \( \partial B_u \cap \partial B_w = \emptyset \) and \( \partial B_s = \partial B_p \cap \partial B_w \) with \( \partial B_p \cap \partial B_w = \emptyset \), we define given boundary values with

\[
\begin{align*}
\mathbf{u}_s &= \mathbf{u}_f \quad \text{on } \partial B_u, \\
\mathbf{t} &= \mathbf{n} = \mathbf{t}_s \quad \text{on } \partial B_u, \\
\mathbf{p} &= \mathbf{p} \quad \text{on } \partial B_p, \\
e_f &= n_s w_{FS} \mathbf{n} = \mathbf{t}_f \quad \text{on } \partial B_w.
\end{align*}
\tag{14}
\]
where $\mathbf{u}_s$ is the displacement of the solid body, $\mathbf{t}$ represents the mechanical traction, $\sigma = \sigma_s + \sigma_f$ defines the sum of the stress tensors, $\mathbf{n}$ denotes the outward normal vector, $e_F$ is the fluid volume flux and the overbar characters denote prescribed (given) functions on the boundaries.

By employing the constitutive relations, the biphasic mixture body can be described by a two-field problem with the set of unknown quantities $\mathcal{R}$, where

$$\mathcal{R} = \{\mathbf{u}_s, p\}. \quad (15)$$

The displacement $\mathbf{u}_s$ of the solid body can be determined by the balance of momentum equation for the mixture

$$\sum_{a} \int_{B_s} (\text{div} \sigma^a + \rho^s \mathbf{b} + \mathbf{p}) \, dv = \int_{B_s} [\text{div} \sigma^a + (\rho^s + \rho^f) \mathbf{b}] \, dv = 0 \quad (16)$$

and the fluid pressure $p$ can be determined by the balance of mass equation for the mixture

$$\sum_{a} \int_{B_s} (n^a + n^a \text{div} \mathbf{X}_a^s) \, dv = \int_{B_s} [(n^s)^T + (n^f)^T + n^s \text{div} \mathbf{X}_s^f + n^f \text{div} \mathbf{X}_f^s] \, dv = 0. \quad (17)$$

Using the relation $(n^s)^T + (n^f)^T = n^F \cdot \mathbf{w}_{FS}$ (cf. Ricken et al. 2010), the balance equation of mass for the mixture (17) can be rewritten in the form

$$\int_{B_s} \text{div}(n^F \mathbf{w}_{FS} + \mathbf{X}_a^s) \, dv = 0, \quad (18)$$

where we make use of the divergence theorem. Equations (16) and (18) form a system of equations in the strong form, where the constitutive relations (5) with (6)–(10) for the stress tensor $\sigma$, and (12) with (13) for the seepage velocity $n^F \mathbf{w}_{FS}$ must be taken into account for the solution. To facilitate the numerical treatment, we generate weak formulations within the framework of a standard Galerkin procedure. The mixture balance of momentum equation (16) is multiplied with the weighting function $\delta \mathbf{u}_s$, and the mixture balance of mass equation (18) is multiplied with the weighting function $\delta p$. As a result, the set of equations in the current and reference configurations can be stated as follows:

- **Balance of momentum equation for the mixture:**

$$\int_{B_s} \mathbf{u}_s \cdot \text{grad} \mathbf{u}_s \, dv - \int_{B_s} \mathbf{b} \cdot \delta \mathbf{u}_s \, dv - \int_{\partial B_s} \mathbf{t} \cdot \delta \mathbf{u}_s \, da$$

$$= \int_{B_s} \mathbf{p} \cdot \text{Grad} \mathbf{u}_s \, dv - \int_{B_s} \rho I_s \mathbf{b} \cdot \delta \mathbf{u}_s \, dv$$

$$- \int_{\partial B_s} p_0 \delta \mathbf{u}_s \, da = 0; \quad (19)$$

- **Balance of mass equation for the mixture:**

$$\int_{B_s} -n^F \mathbf{w}_{FS} \cdot \text{grad} \delta p \, dv + \int_{B_s} D_s I \delta p \, dv + \int_{\partial B_s} e \delta p \, da$$

$$= \int_{B_s} -J_s n^F \mathbf{F}_{FS}^T \cdot \text{Grad} \delta p \, dv$$

$$+ \int_{B_s} J_s (\mathbf{E}_s)^T C_s^{-1} \delta p \, dv + \int_{\partial B_s} e_{FS} \delta p \, da = 0. \quad (20)$$

In (19) and (20),

$$\mathbf{t} = t_s + t_f = \sigma \mathbf{n} = (-n^s p I + \sigma^E) \mathbf{n} - (n^F p I) \mathbf{n}$$

represents the overall traction vector for the mixture with $\mathbf{n}$ as the outward unit surface normal on the boundary $\partial B_s$ in the current configuration, whereas the traction vector in the reference configuration reads $\mathbf{P}_0 = \mathbf{P}_{SO} + \mathbf{P}_{FS} = \mathbf{P}_{OS}$, where $\mathbf{P} = J_s \sigma \mathbf{F}_{FS}^T$ is the first Piola–Kirchhoff stress tensor of the mixture and $\mathbf{n}_{OS}$ denotes the outward unit surface normal on the boundary $\partial B_{OS}$. Moreover, $\mathbf{D}_s$ is the symmetric part of the actual spatial velocity gradient $\mathbf{L}_s = \text{grad} \mathbf{X}_s^f$, both related to the solid phase, whereas $(\mathbf{E}_s)^T = \mathbf{F}_{FS}^T \mathbf{D}_s \mathbf{F}_{FS}$ denotes the material time derivative of the strain rate tensor.
the Green–Lagrange strain tensor $E_S = (C_S - I)/2$ of the solid phase. We also use the properties $dv = J_SdV_{0S}$ and $du = J_SdF_{0S}$, regarding volume and surface elements, respectively. On the Neumann boundary, we consider a fluid mass efflux with $\dot{e}_F = n^T \omega_{FS} \cdot n$ in the current configuration and $e_{0S}^F = n^T F_S^* \omega_{FS} \cdot n_{0S}$ in the reference configuration.

We employ elements of the Taylor–Hood type, with quadratic shape functions for solid displacements and bi-linear shape functions for saturation pressure. The Newmark method is used for the time discretisation. The structure of the linearised weak form of the field equations is given as

$$\langle \delta \Phi \rangle^T (\mathbf{D} \mathbf{K} + \mathbf{D} \dot{\mathbf{K}})\langle \delta \Phi \rangle^T (\mathbf{R} + \mathbf{R}^{\partial E_0}),$$

where the transposed vector of the variation in the unknown field quantities $\langle \delta \Phi \rangle^T$ multiplied by $\mathbf{D} \mathbf{K}$ and $\mathbf{D} \dot{\mathbf{K}}$ represents the stiffness and damping terms, respectively. The vector of known quantities denoted by $\mathbf{R}$ and $\mathbf{R}^{\partial E_0}$ represents the vector of boundary conditions on the surface $\partial E_0$ of the porous body $E_0$.

In order to determine the set of unknowns $\mathbf{R}$ in (15), we use the weak forms of the balance of momentum equations (see (19) = MO) and the balance of mass equations (see (19) = MA). The vector forms of $\mathbf{D} \mathbf{K}$ and $\mathbf{D} \dot{\mathbf{K}}$ are then given by

$$\begin{bmatrix} \mathbf{D} \mathbf{K} \end{bmatrix} = \begin{bmatrix} \mathbf{MO}_{us} & \mathbf{MO}_{up} \\ \mathbf{MA}_{us} & \mathbf{MA}_{up} \end{bmatrix} \begin{bmatrix} \Delta u_s \\ \Delta p \end{bmatrix}$$

and

$$\begin{bmatrix} \mathbf{D} \dot{\mathbf{K}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mathbf{MA}_{us} & 0 \end{bmatrix} \begin{bmatrix} \Delta u_s \\ \Delta p \end{bmatrix}.$$ 

The vector form of the boundary conditions $\mathbf{R}^{\partial E_0}$ reads

$$\begin{bmatrix} \mathbf{R}^{\partial E_0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{\partial MO} \\ \mathbf{R}^{\partial MA} \end{bmatrix},$$

where the boundary conditions

$$\mathbf{R}^{\partial MO} = p_{0S}, \quad \mathbf{R}^{\partial MA} = \epsilon_{0S},$$

act on the surface $\partial E_0$.

We solve the resulting system of equations using a numerical approximation of the tangent moduli in the FE program FEAP (University of California at Berkeley, CA, USA). Such an approach can be used to implement nonlinear hyperelastic models within commercial FE codes, thus encouraging greater use of such models (Sun et al. 2008).

4. **Representative numerical examples**

We complete numerical simulations reproducing experimental tests reported in the cartilage mechanics literature to demonstrate that the proposed constitutive model can approximate many features of the mechanical response of cartilage. Mechanical tests of articular cartilage are generally completed on specimens cut into simple test geometries, either from the full cartilage thickness (e.g. Holmes et al. 1985; Soltz and Ateshian 2000) or, increasingly, from specific layers through the cartilage thickness (e.g. Bachrach et al. 1998; Elliott et al. 2002). Simple test specimen geometries cut from the specific zones and modelled here for illustrative purposes including strips for tension testing (cf. Elliott et al. 2002; Huang et al. 2005) and for compression testing, both discs (cf. Ateshian et al. 1997; Soltz and Ateshian 1998; Wong et al. 2000) and cubes (Wang et al. 2003; Chahine et al. 2004; Erne et al. 2005).

Using the cartilage mechanics literature as a starting point, we employed a manual iterative procedure (adjusting the material and structural parameters) to determine a set of material parameters for each cartilage zone and to demonstrate salient features of the proposed finite-strain material model and 3D FE implementation by approximating several features of the mechanical response observed experimentally. The resulting material and structural parameters used here agree with those in the literature, and also with the expected trends through the thickness: from the superficial zone, through the middle zone and down to the deep zone, consistent with the accepted layer classification scheme for cartilage (Mow et al. 2005; Athanasiou et al. 2010).

We exercise the three 3D FE specimen geometries (strips, discs and cubes) to replicate published experimental tests using various boundary conditions (simulating different tests) and material/structure parameters (simulating different tissue zones). The combined results from the computational experiments can reproduce representative published experimental results, here demonstrated by the following important mechanical responses of cartilage: tension–compression nonlinearity (Ateshian et al. 1997; Elliott et al. 2002), creep (Soltz and Ateshian 1998; Mow et al. 2005; Mansour 2008) with significant fluid pressure load support (Soltz and Ateshian 1998), variable instantaneous and final Poisson’s ratio (Jurvelin et al. 1997; Wong et al. 2000), non-homogeneous, depth-dependent strain (Erne et al. 2005) and specimen deformation (Wong et al. 2000).

### 4.1. Parameters

Table 1 gives a summary of the required material and structural parameters, the equation number where the specific parameter is first introduced, the values used to approximate the response of the three cartilage layers (superficial, middle and deep zones) and the corresponding units. The material and structural parameters vary through the tissue thickness in accordance with the literature; thus, the modelling approach is inhomogeneous through the tissue thickness (three zones). The constitutive model is also
The fluid cannot be ‘squeezed out’ from the tissue and therefore, zones, respectively, as reported by Responte et al. (2007). 86%, 72% and 67% in the superficial, middle and deep layers, collagen content by dry weight is approximately of trapped intrafibrillar fluid (21%, 19%, 17%) follows that fraction should be 0.19 and 0.17 for the middle and deep reasoning for each layer, the trapped intrafibrillar fluid with tissue deformation. Regarding the exponent on the parameter equation, Superficial zone Middle zone Deep zone Unit
\[ \rho \ \text{FR} \] (1) 1000 1000 1000 kg/m³
\[ \rho \ \text{SR} \] (1) 1000 1000 1000 kg/m³
\[ n_0 \] (13) 0.15 0.25 0.30
\[ j_0 \] (9) 0.36 0.44 0.47
\[ k_0 \] (13) \( 1 \times 10^{-15} \) \( 2 \times 10^{-16} \) \( 1 \times 10^{-16} \) m³/Ns
\[ m \] (13) 3.0 7.0 8.0
\[ \mu \] (7) \( 0.02 \times 10^6 \) \( 0.5 \times 10^6 \) \( 0.6 \times 10^6 \) N/m²
\[ k_1, k_2 \] (10) \( 3.0 \times 10^6, 8.0 \) \( 3.0 \times 10^6, 8.0 \) \( 3.0 \times 10^6, 8.0 \) N/m²,
\[ \alpha_0 \] (6) Aligned to split-line Normal to bone
\[ \rho \] (10) 0.90 0.60 1.0

capable of accepting sample- or patient-specific data structures if available.

We discuss the parameters briefly and put them into context within the cartilage mechanics literature. Both the density of fluid and solid, \( \rho \ \text{FR} \) and \( \rho \ \text{SR} \), are set to 1000 kg/m³ as a first approximation. Regarding the reference solid volume fractions \( n_0 \), it is well established that 68–85% of the tissue is fluid (Mow et al. 2005; Mansour 2008; Athanasiou et al. 2010) and thus 15–32% of the tissue is solid. The water content steadily decreases from the tissue surface (superficial zone) to the subchondral bone (deep zone).

In normal cartilage, a portion of this water (approximately 30%) resides in the intrafibrillar space of collagen, and most of this water is not available for transport under mechanical loading (Mow et al. 2005). If we assume that 25% of the initial fluid in each zone resides in the intrafibrillar space of collagen (i.e. ‘most’ of 30%), then, e.g. in the superficial zone, 0.25(1 – \( n_0 \)) = 0.25(1 – 0.15) = 0.21, meaning that 21% of the total fluid cannot be ‘squeezed out’ from the tissue and \( j_0 \) = \( n_0 \) + 0.21 = 0.15 + 0.21 = 0.36. Following this reasoning for each layer, the trapped intrafibrillar fluid fraction should be 0.19 and 0.17 for the middle and deep zones, respectively. Furthermore, this trend in the portion of trapped intrafibrillar fluid (21%, 19%, 17%) follows that due to the decrease in collagen content through the cartilage layers, collagen content by dry weight is approximately 86%, 72% and 67% in the superficial, middle and deep zones, respectively, as reported by Responte et al. (2007). Therefore, \( j_0 \) is set to 0.36, 0.44 and 0.47 for the superficial, middle and deep zones, respectively.

The initial Darcy permeability \( k_0 \) falls typically in the range of \( 10^{-15} \) to \( 10^{-16} \) m³/Ns (Huang et al. 2005; Mow et al. 2005; Mansour 2008), and the permeability of cartilage changes through the tissue thickness, being highest in the superficial zone and lowest in the deep zone (Mansour 2008). Furthermore, the permeability also varies with tissue deformation. Regarding the exponent on the Darcy permeability \( m \), as cartilage is compressed its permeability decreases (Mansour 2008), and many similar volume fraction functions use a value between 3 and 8 (Huang et al. 2005). Furthermore, the magnitude of the deformation-dependence constant \( m \) should increase with increasing depth from the cartilage surface (Chen et al. 2001; Mow et al. 2005).

The equilibrium shear modulus of cartilage (the composite of matrix and fibre components) is often measured in the range of 0.05−0.25 MPa (see, e.g. Mow et al. 2005 and Athanasiou et al. 2010). Another review of nine independent research studies provides the range of 0.1–7.7 MPa, depending on the species and zone within the tissue (Goreham-Voss et al. 2007). Therein, a particular study by Buckley et al. (2007) measured the shear modulus of neonatal bovine articular cartilage to be 0.25, 1.0 and 1.8 MPa in the superficial, middle and deep zones, respectively. Another study by Wong et al. (2007), wherein the depth-dependent shear strains were measured, confirmed the same trend: the initial shear modulus of solid matrix \( \mu \) increases with depth. In biphasic (vs. triphasic) theory, the Donnan osmotic pressure (which contributes to the equilibrium stress state) is also lumped with the stiffness of the solid matrix, i.e. the osmotic pressure contributes to the tissue stiffness (Soltz and Ateshian 1998).

Values for the fibre fabric magnitude and nonlinearity, \( k_1 \) and \( k_2 \), respectively, are not well understood for cartilage. The collagen diameter and content change with depth (cf. Responte et al. 2007; Potter et al. 2009), hence \( k_1 \) and \( k_2 \) would (most likely) change as a function of tissue zone. Since data are not available to determine this layer-specific dependence, we have estimated \( k_1 \) and \( k_2 \) for the superficial zone and left these parameters constant through the thickness. The values estimated here are different (\( k_1 \) higher, \( k_2 \) lower) than those reported in García and Cortés (2007) for an undispersed strain energy function of the exponential type, but are on the order of those parameters of an external iliac artery associated with the collagen fibre response in Holzapfel et al. (2005b). The collagen content
also vary through the thickness (Responte et al. 2007), but this variation has not been included in the current set of material properties.

The local principal direction of fibre fabric \( \mathbf{u}_i \) is consistent with the literature: the fibres are aligned to the local split-line in superficial zone, randomly oriented in the middle zone and aligned perpendicular to the subchondral bone in the deep zone, cf. Figure 1; Responde et al. (2007). The dispersion \( \rho \) of fibre fabric about \( \mathbf{u}_i \) is also consistent with the literature: in both the superficial and deep zones, the fibres are relatively well aligned while the fibres are more dispersed in the middle zone (Responte et al. 2007).

### 4.2. Tension–compression nonlinearity

Tension–compression nonlinearity is a term used in the literature to describe a response of cartilage under different loads: the stiffness of cartilage in tension is generally one to two orders of magnitude higher than that in compression (Huang et al. 2003, 2005). To demonstrate that the proposed constitutive model can reproduce these experimental observations, we exercised an FE model of a cartilage strip in uniaxial tension for both the superficial zone (with the fibres aligned in the loading direction, i.e. parallel to the local split-lines) and the middle zone (with a random fibre orientation). In compression, we exercised an FE model of a cartilage disc in confined compression for the superficial zone with the fibres aligned perpendicular to the loading direction.

The half-symmetry, tension strip was 1.0 mm wide, 0.25 mm thick and 5.0 mm long (in the loading direction), consistent with the experimental tests completed by Elliott et al. (2002). We applied a linear displacement ramp to one end of the tensile strip at a rate of 8.33 \( \times 10^{-3} \) mm/s until an applied stretch of 1.14 was reached. The quarter-symmetry, unconfined compression disc was 0.35 mm thick with a radius of 3.18 mm. We applied a linear displacement ramp to the top surface of the disc at a rate of \(-0.35 \times 10^{-6}\) mm/s up to an applied stretch of 0.50. We calculated the applied stretch from the applied displacements and extracted the normal component of the Cauchy stress from the models in the loading direction.

Figure 3 shows the comparison of the FE modelling results directly to experimental results adapted from Elliott et al. (2002) (Figure 4 therein) for tension and Ateshian et al. (1997) (Figure 3 therein) for compression.

The simulations closely match the experiments for tensile testing of both the superficial zone (with fibres aligned parallel to the load) and the middle zone. Both tension simulations fall well within one standard deviation of the stress determined in the experiment for the entire tensile stretch. Similarly, compression testing of the confined disc closely matches simulation to experiment.

Huang et al. (2005) also provide tension–compression data on equilibrium stresses versus stretch (0.50–1.12, Figure 8 therein). These data give results for the superficial and middle zones that are both parallel and perpendicular to the local split-line direction. The model proposed here can reproduce the experimental data in the superficial zone when parallel to the split-line direction, but will be in error (too soft) when tested perpendicular to the split-line direction. For the middle zone, the current modelling approach is not able to separate these two cases (parallel and perpendicular to the local split-line), as the fibres are assumed to be element-wise random in this zone.

### 4.3. Creep displacement and fluid pressure load support

Both creep displacement and stress relaxation are responses of cartilage under compressive load, and are intimately connected to load supported by fluid pressure. As a representative example, we focus here on creep displacement. When a constant compressive force is applied to a cartilage sample, the resulting axial compression (displacement) will increase nonlinearly: rapidly at first, then progressively more slowly until an equilibrium compression is reached. During this process, the fluid pressure generally supports a significant portion of the total load until the fluid pressure decays towards zero.

To demonstrate that the proposed constitutive model can reproduce these observations, we exercised an FE model of a cartilage disc in confined compression for the middle zone (with the fibres distributed in an element-wise random manner). The quarter-symmetry, confined compression disc was 1.5 mm thick with a radius of 3.26 mm. We applied a constant compressive force of \(-24\) N (distributed) to the top surface of the disc for up to 10,000 s. Both Soltz and Ateshian (1998) and Mansour (2008) provide good descriptions of this test.

We calculated the applied stretch from the applied displacements and extracted the Cauchy stress from the models in the loading direction. Similarly, we extracted the fluid pressure and defined the fluid pressure load support ratio \( R_i^F \) at a tissue point in the \( i \)-direction as

\[
R_i^F = -\frac{\sigma_i}{\sigma_i^T} = -\frac{p}{p + \sigma_i^S},
\]

where \( \sigma_i^T \) is the \( ii \)-component of the total Cauchy stress tensor and \( \sigma_i^S \) is the \( ii \)-component of the effective solid Cauchy stress tensor \( \sigma_i^S \). A similar definition for the fluid pressure load support can be found in Soltz and Ateshian (2000).

Figure 4 shows the comparison of the FE modelling results for both the creep displacement versus time and the fluid pressure load support versus time directly to results adapted from experiments. In terms of the creep displacement, we extrapolated the experimental results of Ateshian et al. (1997) from approximately 1600–2000 s under the condition that at this point in the test the ‘surface
creep displacement rate was again less than 0.01 μm/s’ (Figures 4 and 5 therein). We compared the fluid pressure load support to results adapted from Soltz and Ateshian (1998; Figure 5(b) therein).

The creep displacement results from the simulation (Figure 4(a)) closely match those measured from the experiment (Figure 4(b)). Focusing now on the interstitial fluid pressure load support, both the FE and experimental results show a reasonable match (cf. Figure 4(c) vs. (d)). In the experiments, the interstitial fluid pressure supported more than 90% of the total stress for durations as long as 404 ± 229 s (mean ± SD, n = 7) during confined creep compression (Soltz and Ateshian 1998). As shown in the simulation, the interstitial fluid pressure supported more than 90% of the total stress for 160 s, which is just outside one standard deviation in the experiment.

4.4. Apparent Poisson’s ratio as a function of time
Cartilage also exhibits a time-dependent apparent Poisson’s ratio during deformation. The apparent Poisson’s ratio is not a material property, but a measure of the global tissue deformation. The apparent Poisson’s ratio is defined as

$$v_0(t) = -\frac{\epsilon_l(t)}{\epsilon_a(t)},$$

where $\epsilon_a(t)$ is the axial strain and $\epsilon_l(t)$ is the lateral strain, both measured as a function of time. A similar definition for the apparent Poisson’s ratio can be found in, e.g. Elliott et al. (2002).

When a compressive axial displacement (strain) is applied to a cubic cartilage sample the lateral strain can be measured and varies with time and strain rate. Since both the fluid and solid matrix are intrinsically incompressible (cf. e.g. Bachrach et al. 1998), and the solid matrix has a
very low permeability, the tissue should appear instantaneously incompressible and remain so for up to hundreds of seconds (Pierce et al. 2009). If the tissue is subsequently allowed to come to an equilibrium state, the apparent Poisson’s ratio in compression should be much lower, and will depend on the tissue species, location and zone.

To demonstrate that the proposed constitutive model can reproduce these effects, we exercised an FE model of a cartilage cube in unconfined compression for the deep zone, with the fibres aligned parallel to the loading direction. The quarter-symmetry, unconfined cube was 0.9 mm thick (the tissue through-thickness direction). We applied a linear displacement ramp to the top surface of the cube at a rate of $2 \times 10^{-3}$ mm/s up to an applied stretch of 0.8 (20% compressive strain), then held the displacement constant for 4000 s. The specimen geometry and loading are consistent with the experimental tests completed by Chahine et al. (2004).

We calculated the (applied) axial strain from the applied displacements and lateral strain from the resulting displacements, both as a function of time. The FE modelling results for the apparent Poisson’s ratio versus time are shown in Figure 5.

As shown in Figure 5, the cartilage tissue is instantaneously isochoric (i.e. apparent Poisson’s ratio equals 0.5). This compares well with the experimental results of Wong et al. (2000) who measured the instantaneous Poisson’s ratio of adult cartilage as $0.49 \pm 0.08$ (mean $\pm$ SD, $n = 14$), thus confirming the assumption of incompressibility for this tissue. Independently, Bachrach et al. (1998) confirmed experimentally that fluid-saturated cartilage was incompressible under hydrostatic pressure.

Furthermore, in the FE simulation, the tissue remains essentially incompressible (cf. $0.49 \pm 0.08$) for more than 200 s, an observation also consistent with experimental results. Due to the very low permeability of the tissue, this amount of time is required to allow an appreciable portion of the fluid to seep out from the tissue, so the saturated

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Figure 4. Confined compression creep test of a cartilage disc removed from the middle zone, and comparison with experimental results adapted from Ateshian et al. (1997) and Soltz and Ateshian (1998): (a) creep displacement from simulation; (b) creep displacement from experiment (Ateshian et al. 1997); (c) local fluid pressure load support ratio (26) from simulation; (d) fluid pressure load support ratio from experiment (Soltz and Ateshian 1998).
tissue appears compressible (while both the solid and the fluid remain isochoric (Mansour 2008; Pierce et al. 2009)). At equilibrium, the FE simulation predicts an apparent Poisson’s ratio of 0.32. Wong et al. (2000) also measured the equilibrium apparent Poisson’s ratio experimentally, and determined that it was significantly higher for the adult bovine tissue (0.26 ± 0.11) compared to both the foetal (0.09 ± 0.02) and calf (0.11 ± 0.03) cartilage. Jurvelin et al. (1997) studied the equilibrium behaviour of cartilage discs in unconfined and confined ramp-stress relaxation tests and used this experimental evidence to indirectly estimate the apparent Poisson’s ratio for the articular cartilage. The mean value for Poisson’s ratio obtained from the optical analysis was 0.185 ± 0.065 (mean ± SD, n = 9). Both these experimental results demonstrate that the simulated apparent Poisson’s ratio determined in the simulation is reasonable, while also showing the variability in experimental results documented in the cartilage mechanics literature.

### 4.5. Depth-dependent strain

Under confined or unconfined uniaxial compression cartilage will develop an inhomogeneous, depth-dependent strain field in the solid matrix, with the surface zone experiencing the most severe compaction. This inhomogeneous strain field implies that a global applied strain does not characterise the actual compressive strains in each tissue zone.

To demonstrate that the model can reproduce these effects, we exercised an FE model of a full-thickness (three distinct zones) cartilage cube in unconfined compression, with uniaxial compression aligned perpendicular to the articular surface (the 3-direction of the simulation). The quarter-symmetry, unconfined cube was 2.5 mm thick (the tissue through-thickness direction), with the individual layers measuring 0.5 mm (20% of the total thickness), 1.25 (50%) and 0.75 mm (30%) for the superficial, middle and deep zones, respectively. We slowly applied a linear displacement ramp to the top (superficial) surface of the cube until an applied stretch of 0.95 (5% compressive strain), then the specimen was allowed to equilibrate for 1200 s. After equilibration, we compressed the specimen with a linear displacement ramp to a total applied stretch of 0.94 (6% compressive strain, Δε = 1%) in 1200 s. The specimen geometry and loading match with that from the study of Erne et al. (2005).

In order to facilitate comparison between the FE modelling results and the experimental results (Erne et al. 2005), we define (E_{33})_{1,6} as the difference in the normal, axial component (the 33-component) of the Green–Lagrange strain tensor at 6% total compression (after 1200 s) and the Green–Lagrange strain at 5% total compression (after equilibration), reflecting changes in the strain distribution due to 1% incremental loading after the off-set strain application of 5%. We completed FE simulations with the three-layer fibre-reinforced model, as well as a three-layer isotropic model (identical except the structural fibre fabric reinforcement and fibre-dependent permeability are removed). Figure 6 shows the comparisons of modelling results for both the fibre-reinforced and non-fibre-reinforced (isotropic) models.

Erne et al. (2005) employed electronic speckle pattern interferometry to capture full-field stain maps over entire patellofemoral cartilage cross sections undergoing unconfined uniaxial compressive strain increments of 0.66% as the difference in the normal, axial component (the 33-component) of the Green–Lagrange strain tensor at 6% total compression (after 1200 s) and the Green–Lagrange strain at 5% total compression (after equilibration), reflecting changes in the strain distribution due to 1% incremental loading after the off-set strain application of 5%. We completed FE simulations with the three-layer fibre-reinforced model, as well as a three-layer isotropic model (identical except the structural fibre fabric reinforcement and fibre-dependent permeability are removed). Figure 6 shows the comparisons of modelling results for both the fibre-reinforced and non-fibre-reinforced (isotropic) models.

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Wong et al. (2000) measured the lateral expansion of full-thickness cartilage cylinders undergoing unconfined uniaxial compression (5%, perpendicular to the articular surface). The experiments of Wong et al. (2000) used frictionless loading platens, while the simulations shown here (Figure 6) displaced the top surface with frictionless contact while the bottom surface was fixed (to simulate the effect of attachment to the subchondral bone present in the experiments of Erne et al. (2005)). Nonetheless, in the experiments of Wong et al. (2000), the superficial zone shows the least lateral expansion, consistent with the global deformation of the fibre-reinforced model.

For an additional comparison, Figure 6(c)--(d) shows the fluid pressure distribution (due to $\Delta \varepsilon = 1\%$ incremental loading in 1200 s subsequent to the equilibrium at 5% off-set strain) for both the fibre-reinforced and non-fibre-reinforced (isotropic) models. At this instant of time, the pressure in the centre of the fibre-reinforced model (2.9 $\times$ 10$^{-3}$ MPa) is 4.5 times greater than that in the isotropic model (6.5 $\times$ 10$^{-4}$ MPa). The fibre fabric-dependent permeability modulates the standard behaviour of the poroelastic formulation and acts to maintain fluid pressure beneath the contact surface, and thus enhances load support. It has been noted in the cartilage mechanics literature that the arrangement of the collagen fibre network plays an important role in directing fluid flow in order to optimise tissue function, (cf. Federico and Herzog 2008).

5. Discussion

FE simulations of experimental tests reported in the cartilage mechanics literature show that the proposed constitutive model and numerical formulation can approximate several unique features of the mechanical response of cartilage. It is important to note that we made no direct effort to match the multitude of test results reported in the literature, nor to directly curve-fit specific tests. In the cartilage literature, experimental results are reported using a wide range of species, ages, tissue sites, tissue zones and even states of degeneration, all factors that are known to cause significant variability in the results (Athanasiou et al. 1991; Joshi et al. 1995). Beyond the specific experimental tests selected here, many other tests reported in the cartilage mechanics literature should also be studied numerically, e.g. fluid pressure load support in unconfined compression (Park et al. 2003) and the apparent Poisson’s ratio in tension (Elliott et al. 2002).

The FE simulations shown here use zone-specific sets of material and structural parameters for human articular cartilage consistent with experimental values published in the literature. Previous 3D FE simulations have included inhomogeneous material properties derived from experiments: e.g. depth-dependent Young’s modulus and Poisson’s ratio of the drained non-fibrillar matrix, fibrillar tensile modulus, collagen content (of horizontally oriented fibrils), permeability and initial void ratio based on the results reported in the literature (Li et al. 2000); depth-dependent fixed charge density and tissue solid content from the literature (Chen et al. 2006); spatial collagen
content, fibril orientation and proteoglycan content determined from polarised light microscopy (PLM; Julkunen et al. 2007); depth-dependent collagen content, fibril orientation, fixed charge density and water content from a combination of PLM and biochemical analysis (Julkunen et al. 2008); zone-dependent fibre orientation and moduli, nonfibrillar porous matrix moduli and permeability (moduli estimated using uniaxial tension data; Li et al. 2009) and depth- and split-line-dependent collagen fibril orientation, as well as depth-dependent collagen density and fluid fraction from previous studies (Mononen et al. 2012).

In the proposed model, we incorporate the collagen fibre fabric response using an element-wise principal direction and a measure of fibre dispersion about this direction (Holzapfel et al. 2005a, 2005b; Lilledahl et al. 2011). Other published models accounted for fibre reinforcement either implicitly (e.g. the conewise linear elasticity theory (Soltz and Ateshian 2000) or explicitly. Modelling approaches that explicitly include fibre contributions are a single principal fibre direction (Pierce et al. 2009), 3 orthogonal fibre bundles (Li et al. 2000; Korhonen et al. 2003), up to 4 primary bundles and 13 (randomly oriented) secondary bundles (Wilson et al. 2004, 2005, 2007) and continuous fibre angular distributions (Ateshian et al. 2009).

The permeability tensor introduced here accounts for transformations (affine at the quadrature point level) of the tissue fluid transport properties (i.e. (13) via dependence on $F, a_0$ in the current configuration) and isotropic deformation dependence on the compaction of pore space (13) and includes a condition to restrict tissue compaction (7)–(9). Ateshian and Weiss (2010) provide a general framework for formulating constitutive relations for the anisotropic, strain-dependent hydraulic permeability tensor in deformable porous media that includes the possibility of strain-induced anisotropy.

The proposed model captures changes in permeability due to increased fibre alignment under finite strain, but the direct strain dependence of the permeability on the deformation is isotropic. Reynaud and Quinn (2006) determined the anisotropic hydraulic permeability of the middle zone of articular cartilage under compression using a combination of experimental measurements and FE modelling. Therein, the permeability of the middle zone was isotropic under 0–10% static compression, but became progressively more anisotropic under increasing static compression such that a 10-fold difference between the axial and radial directions was evident under 20–40% static compression.

We do not incorporate viscoelastic effects, which are known to exist in both the poroelastic solid matrix and the fibre fabric (as demonstrated experimentally by Huang et al. (2001)) into the proposed constitutive model. Such viscous solid effects have been included in other cartilage models (e.g. Li and Herzog 2004; Wilson et al. 2004; Pierce et al. 2009).

We assume that the total ion concentration of the tissue remains constant (i.e. constant electrolytic conditions) and lump the Donnan osmotic pressure with the stiffness of the solid matrix. Thus, the model cannot capture the effects of osmotic swelling (cf. Ateshian et al. 2004; Chahine et al. 2004 and references therein), which may induce prestraining of the collagen fibres. Furthermore, under large compression when abundant fluid is expelled, the fixed charge density may significantly increase, therefore increasing the osmotic pressure, even though total ion content remains constant. The proposed model does not capture such effects. Inhomogeneity and tissue swelling have been incorporated into other cartilage models, which also include some description of collagen fibre reinforcement (Wilson et al. 2005, 2007).

The dispersion parameter $\rho$ could be adjusted to account for collagen fibrillation resulting from degenerative changes, or correlated to the state of degeneration in the tissue, in order to study disease progression, e.g. osteoarthritis. In fact, a damage model that links increasing degeneration to reductions in $\rho$ (increased collagen fibrillation) and $\mu_S$ (reduction in proteoglycan content), and increases in $n^F$ (increased fluid content) could be proposed. These issues, which will increase the complexity of cartilage modelling, can be addressed in subsequent formulations.

In short, we have proposed a 3D finite-strain model capable of simultaneously addressing both solid (reinforcement) and fluid (permeability) dependence of the tissue’s mechanical response on the collagen fibre fabric. We represent fibre reinforcement as an inhomogeneous, dispersed fabric, where the fluid permeability is intimately dependent on this fabric and a fixed intrafibrillar water portion. The proposed approach, in which the material parameters are structurally motivated and have direct physical interpretation, facilitates 3D patient-specific simulations and can be employed to implement high-resolution morphological data in a computational setting (see, e.g. Pierce et al. 2010 and Lilledahl et al. 2011). We also note that a more general patient-specific model should additionally include (at least) patient-specific, proteoglycan-related osmotic pressure. Nonetheless, we propose a step towards studying the functional response of patient-specific structural and diffusional properties in cartilage.

Improved constitutive models for simulating soft tissue deformation are increasingly valuable in many areas of biomedical engineering. Such constitutive models for articular cartilage (combining advanced imaging and computational biomechanics) enable studies of, e.g. fundamental structure–function relationships, surgical interventions, cartilage and joint integrity, disease progression, cartilage replacements and imaging data analysis, and provide insight into microphysical (mechanobiological) cellular stimuli.
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